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# The Building Materials Durability Model (BMDM)

## A Comparative Model for Service Life Factors Affecting Materials Selection

by

K.D. Hjelmstad, D.A. Lange, F.V. Lawrence, I.D. Parsons, R.F. Quattrone, J.C. Trovillion, and D.M. Bailey

Construction materials account for a major portion of building costs, affecting the costs of initial construction, continued maintenance, habitability, and eventual demolition of a building. The initial selection of a construction material may depend on a number of complex and often intangible factors, but the total initial and long-term costs of using any building material system is one of the most important parameters for U.S. Army planners and budgeteers. The objective of this research was to investigate the feasibility of using modeling and simulation techniques as the basis for a tool to help Army facility designers and engineers select materials for optimal life-cycle economy and serviceability.

The researchers developed algorithms representing various materials durability parameters and environmental stimuli, and integrated these into a prototype Building Materials Durability Model (BMDM). This report discusses the logic and assumptions used in developing BMDM, and describes the results of hypothetical simulations on token building elements tested in two different climates. It is concluded that BMDM could be extended, refined, and used with Monte Carlo simulations to reliably project and compare materials durability costs.

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## Foreword

This research was conducted for Headquarters, U.S. Army Corps of Engineers under Project 4A162784AT41, "Military Facilities Engineering Technology"; Work Unit FM-CM5, "Optimal Materials and Products Selection in Concurrent Engineering." The technical monitor was Rodger Seeman, CEMP-EA.

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COL James T. Scott is Commander and Acting Director of USACERL, and Dr. Michael J. O'Connor is Technical Director.

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# 1 Introduction

## Background

Construction materials account for a major portion of building costs, affecting the costs of initial construction, continued maintenance, habitability, and eventual demolition of a building. The initial selection of a construction material may depend on a number of complex and often intangible factors, but the total initial and long-term costs of using any building material system is one of the most important parameters. Materials with the lowest initial cost often result in such high maintenance costs that any initial cost savings can be erased much earlier than expected. A key to cost reduction through materials selection is knowledge of the comparative durability of various building materials, and how their durability affects the total building's total life-cycle cost.

The service life of a structure is usually considered during design only through the specification of the return period of the most extreme environment the structure is expected to endure. For example, a structure may be designed to withstand a 100-year earthquake or wind storm. The selection of materials typically is based on the notion that, with some safety factor built in, the structure will resist the extreme event in a virgin state. A good engineer considers the implications of material durability and maintenance in the design of a structure even though current Army design requirements do not explicitly insist on it. The current state of the art in engineering design for durability is based on rather coarse decision points such as "steel corrodes while plastic does not," or "concrete scales while steel does not." Such anecdotal knowledge does not really help the engineer evaluate the tradeoffs in materials selections because it is too superficial. The significance of materials selection on overall Army operational costs can be seen in corrosion cost figures, for example, which exceed \$300 million annually (Hahin 1977). Corrosion costs control are often most easily reduced during facility design through the selection of alternative materials.

The lack of long-term experience with promising new materials tends to inhibit application of new materials that might significantly reduce long-term facility costs. Facility designers and specifiers need a model that can predict the durability of various building materials as used for a specific application at a specific location. An effective model of this type also would analyze what role or priority materials

durability should play in the selection process. Such a model, when used during concept design in a concurrent engineering collaborative decision environment, would enable engineers to accurately and efficiently compare the life-cycle costs of conventional materials to alternative ones. The model could help the Army maintain the life expectancy of structures at a lower life-cycle cost.

The U.S. Army Construction Engineering Research Laboratories was tasked by Headquarters, U.S. Army Corps of Engineers, to develop a Building Materials Durability Model (BMDM).

## **Objective**

The objective of this work was to investigate concepts for optimal selection of building materials, for possible incorporation into a decision-support tool for facility designers using an agent-based collaborative design network.

## **Approach**

The researchers investigated modeling techniques that consider predictive, long-term materials performance in service, maintenance and repair (M&R) life-cycle implications, and differing construction environments.

A probabilistic modeling approach was used. Damage models were developed for several common construction material systems, both for structural elements and cladding elements. The model's algorithms were embedded into a spreadsheet software program for desktop computer. Token building elements were subjected to differing environments for a number of iterations in a computer-driven Monte Carlo simulation, and material performance was evaluated. Repair and failure criteria were evaluated after each iteration, and the accumulated cost of durability calculated. A presentation format for durability comparison of different construction materials was developed. Finally, BMDM was applied to four materials—two structural materials and two cladding materials. Durability costs, in terms of net present value, were calculated for two different natural environments.

## **Mode of Technology Transfer**

This work will feed into the Architect's Associate program, USACERL's concurrent engineering research initiative. The Architect's Associate technology transfer plan will

include the formation of cooperative research and development agreements (CRaDAs) for transfer of applications and development of an agent collaboration language. The agent collaboration language will be proposed for inclusion by ANSI and ISO\* in the Product Data Exchange Specification (PDES) and the Standard for the Exchange of Product Model Data (STEP).

### Metric Conversion Factors

U.S. standard units of measure are used throughout this report. A table of metric conversion factors is presented below.

1 in.	=	25.4 mm
1 sq in.	=	645.16 mm <sup>2</sup>
1 lb	=	3.78 L
°F	=	(°C × 1.8) +32

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\* ANSI is the American National Standards Institute; ISO is the International Standards Organization.

## 2 Description and Development of BMDM

### Durability Model Concept

Suppose that it is possible to simulate the evolution of all environmental stimuli that are important to the durability of a building component. Suppose further that it is possible to analytically model the damage processes that can occur in the building component and hypothesize various repair strategies. Given these assumptions, it would be possible to simulate the lifetime of that building component over time within the framework described in Figure 1\*. In this framework the environmental stimuli  $T(t)$ ,  $\theta(t)$ , and  $c(t)$  are specified as explicit (possibly random) functions of time. The component includes a number of descriptors, including its material properties and its geometric configuration. The quantities  $x$  represent a collection of damage variables recording loss of material, change in material property, loss of aesthetic appeal, and other factors that evolve according to the damage rate functions  $f$ . With an idea of the component's initial cost, the criterion  $h(x) \leq \gamma$  can be established to indicate failure or to trigger repair and establish the means to specify a repair strategy and accumulate durability cost (i.e., initial cost plus the sum of the repair costs).

During each time period, the amount by which the material service properties have degraded is determined according to the damage model. The degraded building element is checked for compliance with specific performance and serviceability criteria. If the component has failed during that time period, its life is over. The model then computes the component's lifetime and durability costs. If the structure has not failed, but instead a repair has been triggered, then the repair is executed, the durability cost revised, and the element is ready to enter the next time period. If the element meets all criteria then it simply passes to the next time period. The simulation continues until the design life has expired or the component has failed.

Clearly, the success of this durability model depends upon how well the environmental factors can be simulated and how well the damage processes can be modeled and repair strategies defined. However, the paradigm can be implemented with rather primitive environmental and damage models providing that more sophisticated models can be put in place of the simple ones as they become available. In this work some

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\* Figures are found at the end of their corresponding chapters.

fairly simple models of damage are adopted to demonstrate the synergy of the coupled approach.

## Environmental Stimuli

Over its lifetime, a material component will be subjected to many environmental factors, and these factors will generally fluctuate with time. In this model, the BMDM assume that the changes in these environmental stimuli over time are known. Three environmental effects will be considered here: the ambient temperature  $T(t)$ , the ambient moisture  $\theta(t)$  due to humidity and precipitation (expressed as a fraction of saturation), and the concentration of aggressive chemicals or agents of decay  $c(t)$  (expressed as a fraction of the maximum concentration). Different environments can be modeled by assigning different histories for these three stimuli. Two hypothetical environments will be considered: tropical (possibly indicative of Guam) and continental United States (possibly indicative of Chicago).

Temperature, humidity, and chemical concentration generally fluctuate according to daily and seasonal changes. For the current purpose, the monthly averages of these stimuli are used. When daily fluctuations are important, as they would be in a freeze-thaw model, they will be accounted for explicitly in damage models. Regular seasonal changes will be augmented with random fluctuations to simulate the variability of climatic effects.

The fluctuation of temperature is given by the expression

$$T(t) = T_0 + \Delta T \sin(\pi t / 6) + \delta T \eta(t) \quad [\text{Eq } 1]$$

where  $T_0$  is the average yearly temperature,  $\Delta T$  is half the difference between the highest average monthly temperature and the lowest average monthly temperature,  $\delta T$  is the average fluctuation in temperature in 1 month, and  $\eta(t)$  is a uniformly distributed random number between -1 and 1. The time  $t$  is measured in months. The continental U.S. temperature model has  $T_0 = 40^\circ\text{F}$ ,  $\Delta T = 50^\circ\text{F}$ , and  $\delta T = 10^\circ\text{F}$ . The tropical model has  $T_0 = 85^\circ\text{F}$ ,  $\Delta T = 10^\circ\text{F}$ , and  $\delta T = 5^\circ\text{F}$ .

The fluctuation in moisture is given by a similar expression,

$$\theta(t) = \theta_0 + \Delta \theta \sin(\pi t / 6) + \delta \theta \eta(t) \quad [\text{Eq } 2]$$

where  $\theta_0$  is the average annual moisture content,  $\Delta \theta$  is half the difference between the highest average monthly moisture content and the lowest average monthly moisture,

$\delta\theta$  is the average fluctuation in moisture in 1 month, and  $\eta(t)$  is a uniformly distributed random number between -1 and 1. The continental U.S. moisture model has  $\theta_0 = 0.5$ ,  $\Delta\theta = 0.3$ , and  $\delta\theta = 0.05$ . The tropical model has  $\theta_0 = 0.9$ ,  $\Delta\theta = 0.05$ , and  $\delta\theta = 0.01$ .

The chemical environment also can vary by season, but for current purposes it is modeled simply as a constant average value with random fluctuations,

$$c(t) = c_0 + \delta c \eta(t) \quad [\text{Eq 3}]$$

where  $c_0$  is the average concentration of aggressive chemicals,  $\delta c$  is the expected fluctuation in that average, and  $\eta(t)$  is a uniformly distributed random number between -1 and 1. The continental U.S. model has  $c_0 = 0.7$  and  $\delta c = 0.3$ . The tropical model has  $c_0 = 0.2$  and  $\delta c = 0.1$ .

## General Framework for Damage Models

Damage mechanisms for various materials are generally determined by factors that occur at a microscopic scale. Many of these processes are still poorly understood. It is beyond the scope of the current investigation to attempt to model damage at the microscale. One might eventually replace each of the various damage models used here with more accurate ones, but for current purposes simple models are used to qualitatively reflect the physical processes that are taking place.

Let  $p(t)$  represent a typical damage parameter. Depending upon the mechanism of damage, the evolution of this damage parameter will be governed by one of three basic damage models: *linear growth*, *exponential growth*, or *polynomial growth*.

### Linear Growth Model

A linear growth model does not depend on the current state of damage, but has a rate of growth governed by some function of the environment:

$$\left. \begin{aligned} \frac{dp}{dt} &= f(T, \theta, c) \\ p(0) &= p_0 \end{aligned} \right\} \quad [\text{Eq 4}]$$

For this model, if the environment is constant, damage grows linearly with time:

$$p(t) = p_0 + f(T, \theta, c)t \quad [\text{Eq } 5]$$

Scaling of concrete caused by freeze-thaw cycling is an example of linear growth damage.

### ***Exponential Growth Model***

In an exponential growth model, the rate of change in the damage parameter depends directly upon the amount of damage currently present (sometimes raised to some power  $m$ ):

$$\left. \begin{aligned} \frac{dp}{dt} &= f(T, \theta, c) p(t) \\ p(0) &= p_0 \end{aligned} \right\} \quad [\text{Eq } 6]$$

For this model, if the environment is constant, the damage evolves exponentially with time as follows:

$$p(t) = p_0 e^{f(T, \theta, c)t} \quad [\text{Eq } 7]$$

The solution to the exponential growth model for  $p_0=1$  and constant forcing function  $f=B$  is shown in Figure 2 for different values of  $B$ . Damage to the surface protection provided by paint is modeled as an exponential growth process.

### ***Polynomial Growth Model***

A polynomial growth model represents a diffusion process. The rate of change of the damage parameter is inversely proportional to the amount of damage present (often raised to some power  $m$ ):

$$\left. \begin{aligned} \frac{dp}{dt} &= \frac{f(T, \theta, c)}{p(t)} \\ p(0) &= p_0 \end{aligned} \right\} \quad [\text{Eq } 8]$$

For this model, if the environment is constant, the damage evolves as a polynomial with time as follows:

$$p(t) = \sqrt{p_0^2 + 2f(T, \theta, c)t} \quad [\text{Eq 9}]$$

One of the most important features of this model is that it is usually associated with failure in a finite amount of time. The solution to the exponential growth model for  $p_0=1$  and constant forcing function  $f=B$  is shown in Figure 3 for different values of  $B$ . Corrosion of steel, decay of wood, and static fatigue of concrete are examples of the diffusion (polynomial growth) model.

In each case, the environmental function driving the process will be different. In most cases a four-parameter model was selected, with one tunable parameter for each of the three environmental effects, and one to accurately represent the overall rate of damage. Some of these functional dependencies come from first principles, e.g., Arrhenius's Law (Arrhenius 1907) is used to characterize dependence upon temperature; others, however, are simply plausible heuristic models, e.g., the exponential models often used for moisture and chemical concentration. A typical example of one of these functions is given by:

$$f(T, \theta, c) = D_0[\theta(t)]^\alpha [1+c(t)]^m e^{-\beta/T(t)} \quad [\text{Eq 10}]$$

This type of model "does the right thing" in the sense that higher moisture content drives damage faster, but there is a premium on moisture contents near saturation,  $\theta=1$ , and damage stops if the element dries out. (Not all models have this feature.) Higher concentrations of aggressive chemicals speed up the rate of damage, but damage will occur even if no chemicals are present. High temperatures speed up damage; low temperatures nearly stop the damage process.

### ***Numerical Integration of the Damage Models***

The functions  $f(T, \theta, c)$  driving the damage processes will, in general, not be constant with time. Thus, the rate equations must be integrated numerically. For current purposes the generalized midpoint rule is used. The incremental linear growth model of Equation 4 then takes the form

$$\left. \begin{array}{l} p(t_{i+1}) = p(t_i) + f(T_\alpha, \theta_\alpha, c_\alpha) \Delta t \\ \\ p(0) = p_0 \end{array} \right\} \quad [\text{Eq } 11]$$

where  $\Delta t = t_{i+1} - t_i$ , and the environmental factors are a weighted average of the values at  $t_i$  and  $t_{i+1}$  as follows:

$$\left. \begin{array}{l} c_\alpha = (1-\alpha)c(t_i) + \alpha c(t_{i+1}) \\ \\ \theta_\alpha = (1-\alpha)\theta(t_i) + \alpha\theta(t_{i+1}) \\ \\ T_\alpha = (1-\alpha)T(t_i) + \alpha T(t_{i+1}) \end{array} \right\} \quad [\text{Eq } 12]$$

In general, one should use  $\alpha=0.5$ , i.e., the trapezoidal rule. The other equations are integrated similarly. The exponential growth model has the incremental form

$$\left. \begin{array}{l} p(t_{i+1}) = p(t_i) e^{f(T_\alpha, \theta_\alpha, c_\alpha) \Delta t} \\ \\ p(0) = p_0 \end{array} \right\} \quad [\text{Eq } 13]$$

The polynomial growth model has the incremental form

$$\left. \begin{array}{l} p(t_{i+1}) = \sqrt{p^2(t_i) + 2f(T_\alpha, \theta_\alpha, c_\alpha) \Delta t} \\ \\ p(0) = p_0 \end{array} \right\} \quad [\text{Eq } 14]$$

Throughout this work details of computation are given only for exceptions to the above cases and for computations that help determine the coefficients of the models.

## Cost Modeling

The user can specify an MER strategy in accord with serviceability requirements, safety factors, or other factors affecting the performance of each component. Each

material will be subject to different constraints, but for current purposes the strategy is modeled as follows: the  $v$ th repair act costs  $C_v$  dollars and restores the element properties, as well as certain of the model parameters, to a fraction  $q_v$  of the most recent repaired value or to a fraction  $q_v$  of its original value. An element that fails during a time step must be replaced—it cannot be repaired.

The quality of a component or a repair job will depend on many factors. For example, the quality of a paint job depends upon the quality of the paint, the quality of surface preparation, and the quality of paint application. To simplify this discussion it is hypothesized that the overall quality can be indexed with a single value based on an assumption that a person who buys the best paint would also be inclined to do a good job preparing the surface and applying the paint. In other words, this model simply cannot account for a person doing a sloppy job with good paint. The quality  $q$  will be specified as a number between 0 (poorest quality) and 1 (best quality).

The cost associated with making a repair is also a function of the quality of that repair. Clearly, the best quality repair will cost the most and the worst quality repair the least. The cost of repair is modeled as

$$C(q) = \frac{C_0}{1-q(1-\rho)} \quad [\text{Eq } 15]$$

where  $C_0$  is the cost of the poorest quality repair and  $\rho$  is the ratio of the cost of the poorest quality repair to the cost of the best quality repair. The model yields a “progressive tax” on quality as it nears perfection, but is relatively insensitive to cost at lesser qualities.

The user must specify the initial cost of the component, to facilitate comparison among materials as well as the cost of replacement. In some models certain things can be repaired while others cannot. In those cases, the repair strategy applies only to items that can be repaired. The maintenance scheme requires the specification of the least-expensive repair, the most expensive repair, the initial quality  $q_0$  of the component, and the quality of any subsequent repair  $q_v$  (as well as the criteria that determine the time of repair). These specifications determine the *durability cost*.

The time value of money is considered by finding the present value of a future amount according to the following formula for single payment present worth (SPPW):

$$\text{SPPW} = (1+i)^{-n} \quad [\text{Eq } 16]$$

where  $i$  is the effective discount rate per period (1 month for the current simulations) and  $n$  is the number of periods. If the cost of a repair is  $C_v$  at the time of repair, then the value of that repair at time zero is

$$\hat{C}_v = C_v(1+i)^{-n}$$

[Eq 17]

where  $n$  is the number of periods elapsed since time zero. For the accumulation of durability costs, all costs are referred back to time zero. Therefore, a material requiring repair in 40 years (for example) can be considered less costly in terms of current dollars than a material requiring repair in 30 years. The time value of money—that is, the idea that a dollar today is inherently more valuable than the same dollar next year—may play an important role in comparing a high-maintenance material (such as steel) with a low-maintenance—but initially more expensive—material (such as aluminum).

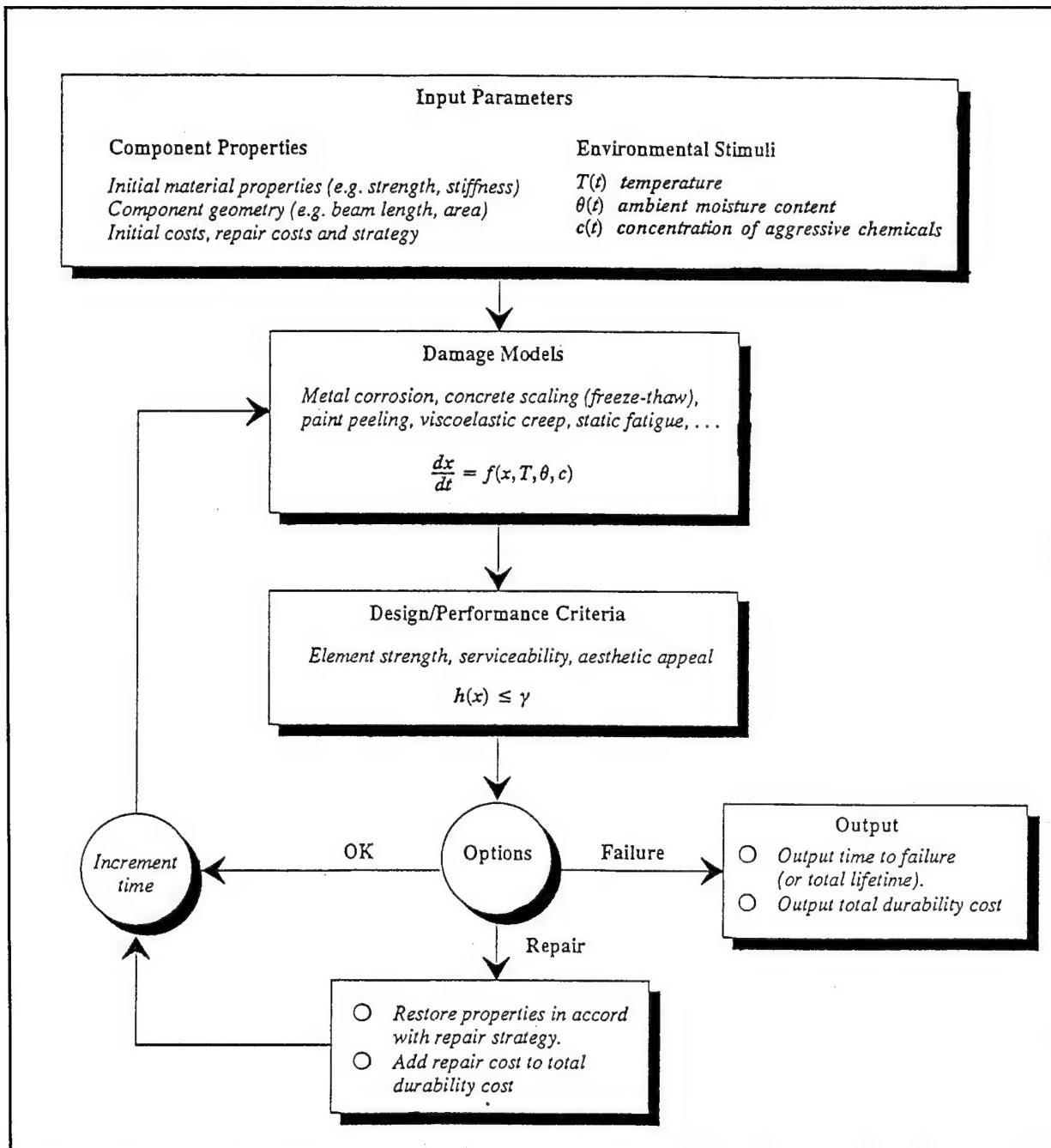


Figure 1. Durability model concept.

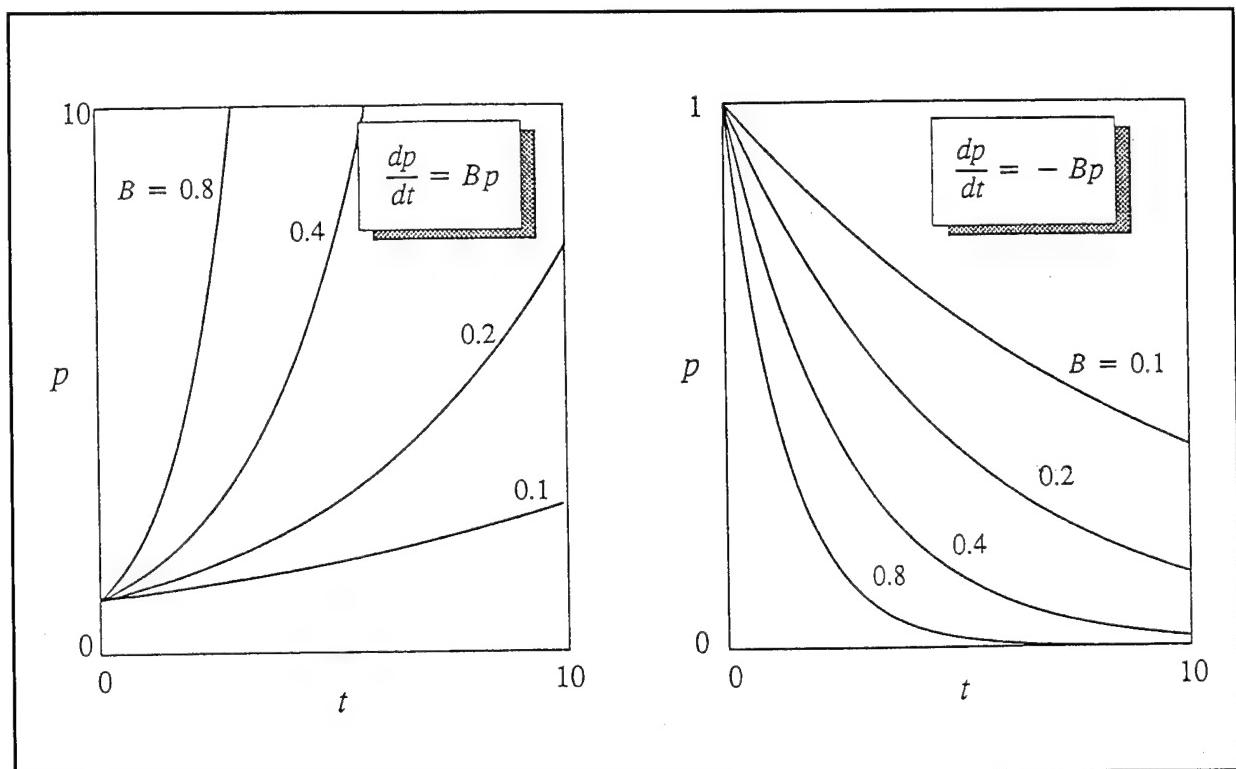


Figure 2. Exponential growth equations with constant coefficients.

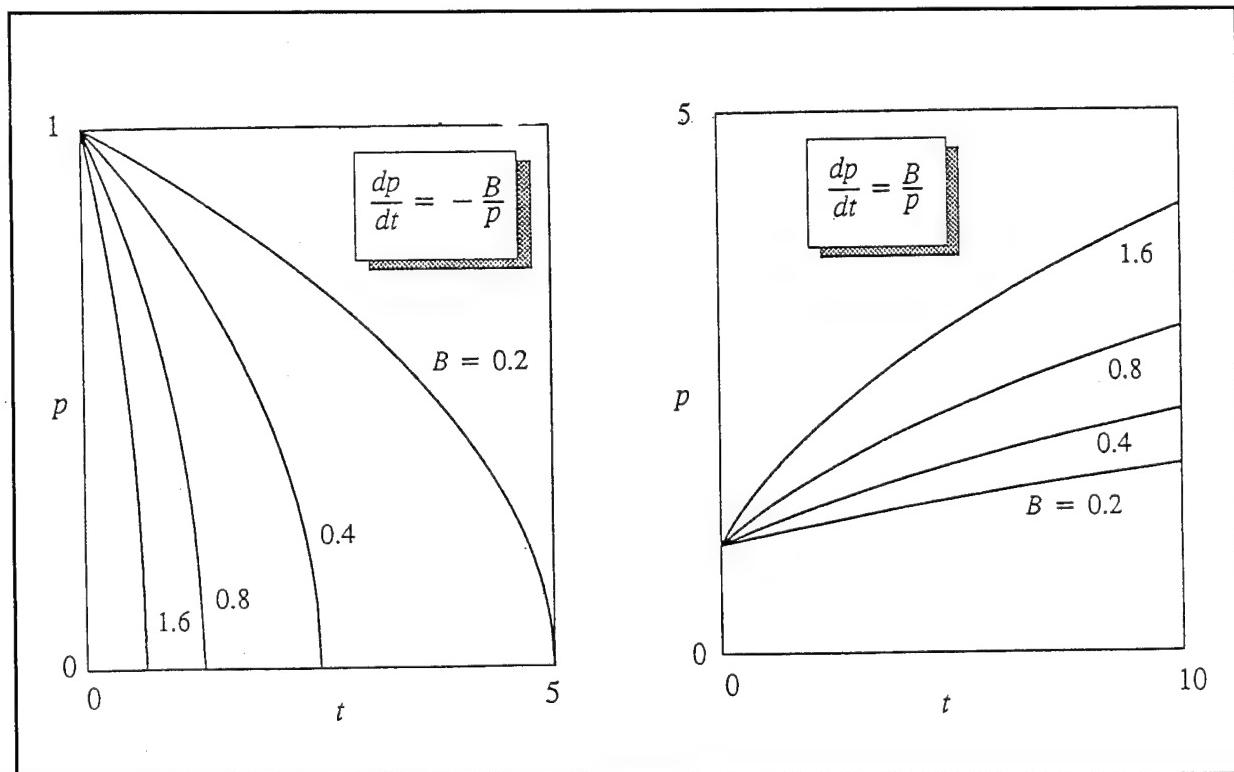


Figure 3. Polynomial growth model with constant coefficients.

### 3 Modeling the Durability of a Structural Element

#### Description of Model Problems

As a model problem, consider the durability of a beam subjected to a loading  $P(t)$  at its midspan, as shown in Figure 4. The materials available for beam construction are reinforced concrete, steel, wood, aluminum, or fiber-reinforced plastic (FRP). Over its lifetime, the beam is subjected to environmental factors that fluctuate with time. As noted in Chapter 2, this problem focuses on three factors:

- ambient temperature  $T(t)$
- ambient moisture  $\theta(t)$  due to humidity and precipitation, expressed as a fraction of saturation
- concentration of aggressive chemicals or agents of decay  $c(t)$ , expressed as a fraction of the maximum concentration.

For the simulations  $P(t)$  takes the following form:

$$P(t) = P_0 + \delta P \eta^2(t) \quad [\text{Eq 18}]$$

where  $P_0$  is the expected load and  $\delta P$  is the amplitude of the fluctuation in the load. As before,  $\eta(t)$  is a uniformly distributed variate (random variable).

The beam must continue to function according to its intended purpose, that is, carry the loading  $P(t)$  without excessive deflection. For each material there are strength and serviceability requirements in terms of cross-sectional properties like the area  $A(t)$  and the moment of inertia  $I(t)$ , which are degrading through various damage mechanisms. Both the onset of failure (or dysfunction) and the M&R cost are considered.

### **Strength Design Criterion**

It is assumed that the beam is designed to have adequate bending strength at time  $t=0$ . The strength of the beam, as a function of time, is designated by  $P_u(t)$ . The initial strength of the beam is thus  $P_u(0)$  and meets the design criterion

$$P_u(0) \geq \phi_o P_{\text{expected}} \quad [\text{Eq 19}]$$

where  $\phi_o$  is the initial factor of safety and  $P_{\text{expected}}$  is the maximum expected applied load. As the properties of the beam degrade over time the strength will decrease, thereby compromising the factor of safety. The *current design factor of safety* is given by

$$\phi_d(t) = \phi_o \frac{P_u(t)}{P_u(0)} \quad [\text{Eq 20}]$$

and define the *current actual factor of safety* is given by

$$\phi_a(t) = \frac{P_u(t)}{P(t)} \quad [\text{Eq 21}]$$

The current design factor of safety will be used as an index for repair decisions (e.g., repair if  $\phi_d < 0.8\phi_o$ ) while the current actual factor of safety will be used to detect failure of the system, i.e.,  $\phi_a < 1$ .

### **Serviceability Design Criterion**

The main serviceability design criterion is the guard against excessive deflection. The initial design will have deflection limits for a variety of reasons, many of which will have little to do with the long-term performance of the beam. As a consequence, a long-term limit to midspan deflection is simply specified as  $w_{ls}$ . The *current serviceability factor of safety* is given as

$$\phi_s(t) = \frac{w(t)}{w_{ls}} \quad [\text{Eq 22}]$$

The current serviceability factor of safety will be used as an index for repair decisions (e.g., repair if  $\phi_s > 1$ ) or to detect functional failure of the system (e.g., the component is dysfunctional if  $\phi_s > 10$ ).

## Durability Model for a Reinforced Concrete Beam

Degradation of the reinforced concrete beam is caused by corrosion of the reinforcement bars, static fatigue of the compressive strength, and scaling of the top surface, as shown schematically in Figure 5. The strength of the reinforced concrete beam at time  $t$  is given by

$$P_u(t) = \frac{4A_s(t)f_y}{L} \left[ h(t) - 0.59 \frac{A_s(t)f_y}{\sigma_o(t)b} \right] \quad [\text{Eq 23}]$$

where  $A_s(t) = n_s \pi [r_o - x(t)]^2$  is the area of  $n_s$  steel bars corroded to a depth of  $x(t)$ ,  $h(t) = h_o - s(t)$  is the beam depth scaled to a depth  $s(t)$ , and  $\sigma_o(t)$  is the concrete compressive strength. The yield strength of the steel,  $f_y$ , and the width of the beam,  $b$ , are taken as constants.

Although reinforced concrete is subject to long-term creep deflections, they are not considered in this problem.

### **Corrosion of Reinforcement Bars**

The beam is reinforced with steel bars of initial radius  $r_o$ . Due to the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ , the bars have a current radius of  $r$ , reduced from the original radius by the amount  $x$ . Figure 6 shows the measure of material loss caused by corrosion.

Corrosion of the reinforcement bars will depend on how much moisture or other chemical agents are present. In lieu of a more sophisticated model of moisture transport, it is assumed that the concrete has pre-existing cracks that allow environmental moisture unimpeded access to the reinforcement bars. The following rate equation describes the loss of bar radius caused by corrosion:

$$\left. \begin{aligned} \frac{dx}{dt} &= \frac{f_c(T, \theta, c)}{x(t)} \\ x(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 24}]$$

The driving function takes the form

$$f_c(T, \theta, c) = D_o [\theta(t)]^m [1 + c(t)]^m e^{-B/T(t)} \quad [\text{Eq 25}]$$

where  $\theta(t)$  is the environmental moisture content as a percentage of saturation,  $c(t)$  is the concentration of the chemical environment, and  $T(t)$  is the temperature (in degrees Kelvin). The parameters  $n$ ,  $m$ ,  $D_0$ , and  $\beta$  are material constants that will be fit to anecdotal data; in other words, all other factors will be held constant while the published variation caused by a single effect are fit.

The solution to the rate can be obtained by numerical quadrature (squaring) to give the loss of radius at time  $t_{i+1}$  in terms of the loss at time  $t_i$  augmented by the corrosion that takes place between those two times:

$$\left. \begin{aligned} x(t_{i+1}) &= \sqrt{x^2(t_i) + 2\Delta t f_c(T_\alpha, \theta_\alpha, c_\alpha)} \\ x(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 26}]$$

The main idea behind this corrosion model is as follows: because corrosion is mainly the result of exposure to moisture, the rate of corrosion should increase as the moisture content increases—and should effectively stop in the absence of moisture. Furthermore, nearly dry conditions should be considerably less corrosive than nearly saturated conditions. Therefore, in Equation 25, the exponent  $n$  is greater than 1. The greater the exponent, the more accentuated this effect will be.

Corrosion is exacerbated by the presence of aggressive chemicals, but it will occur even in the absence of chemicals. The greater the concentration of aggressive chemicals, the faster the corrosion. Therefore, the exponent  $m > 1$ . The greater the exponent, the more accentuated this effect will be.

According to Arrhenius's Law (Arrhenius 1907), the rate of corrosion depends upon the temperature. The material will corrode faster at a higher temperature. The coefficient  $\beta$  controls the rate of corrosion caused by an increase in temperature. Corrosion would be expected to drop to arbitrarily low rates as the temperature drops below the freezing point, and would occur fastest at the top of the temperature range being considered. The maximum environmental temperature may be designated as  $T_f$ . If, at any given instant the rate of corrosion at  $T_f$  were  $\gamma$  times the rate at  $\varepsilon T_f$ , then the exponent  $\beta$  must be given by

$$\beta = \frac{\varepsilon}{1-\varepsilon} T_f \ln \gamma \quad [\text{Eq 27}]$$

For example, if the rate of corrosion is one-fourth as fast at half the maximum temperature of 325 °K, then  $\beta = 450$  °K.

### **Static Fatigue in Concrete**

At loads that cause the compressive stress to approach the strength of concrete, damage is incurred through the mechanism of *static fatigue*. This phenomenon results in a decrease in the strength of concrete over time. Let  $\sigma_o(t)$  be the concrete strength and  $\sigma(t)$  (positive compressive) the applied stress at time  $t$ . Assuming that the material exhibits elastic behavior, one can relate the stress to the applied load though the relationship  $\sigma(t) = P(t)/S$ , where  $S$  is the section modulus of the beam. For current purposes it is hypothesized that the strength degrades according to a power law as follows

$$\left. \begin{aligned} \frac{d\sigma_o}{dt} &= -B\bar{\sigma}_o \left[ \frac{\sigma(t)}{\sigma_o(t)-\sigma(t)} \right]^n \\ \sigma_o(0) &= \bar{\sigma}_o \end{aligned} \right\} \quad [\text{Eq 28}]$$

where  $B$  and  $n$  are the parameters describing the material. Assuming that the applied stress remains constant from time  $t_i$  to  $t_{i+1}$ , Equation 28 can be integrated to give the strength at the current time as

$$\sigma_o(t_{i+1}) = \sigma(t_i) + \left[ [\sigma_o(t_i) - \sigma(t_i)]^{n+1} - (n+1)B\bar{\sigma}_o\sigma^n(t_i)\Delta t \right]^{\frac{1}{n+1}} \quad [\text{Eq 29}]$$

This model will predict a decrease in strength with time, but it can be rather sensitive. Therefore, one must take care to check for material failure within a time step. The time to failure, expressed as a fraction of the current time increment, is given as

$$\tau_f = \frac{[\sigma_o(t_i) - \sigma(t_i)]^{n+1}}{(n+1)B\bar{\sigma}_o\sigma^n(t_i)\Delta t} \quad [\text{Eq 30}]$$

If  $\tau_f < 1$ , then the material has failed during the current time increment. Because one must compute time to failure at every step anyway, Equation 29 can be simplified to

$$\left. \begin{aligned} \sigma_o(t_{i+1}) &= \sigma(t_i) + [(T_f - 1)(n+1)B\bar{\sigma}_o\sigma^n(t_i)\Delta t]^{\frac{1}{n+1}} \\ \sigma_o(0) &= \bar{\sigma}_o \end{aligned} \right\} \quad [\text{Eq 31}]$$

The constants can be fit to data obtained from tests in which the time to failure is recorded for constant stress values. In such an instance, failure is defined as  $\sigma_o(t_f) = \sigma$ , and time to failure is given by

$$t_f = \frac{(\bar{\sigma}_o - \sigma)^{n+1}}{(n+1)B\bar{\sigma}_o\sigma^n} \quad [\text{Eq 32}]$$

Assume that two tests are available. Let  $(\sigma_1, t_1)$  be the applied stress and time to failure of the first test, and let  $(\sigma_2, t_2)$  be the applied stress and time to failure of the second test. The exponent of the model can be determined from these two tests by the following relationship:

$$n \ln \left[ \frac{\sigma_1(\bar{\sigma}_o - \sigma_2)}{\sigma_2(\bar{\sigma}_o - \sigma_1)} \right] = \ln \left[ \frac{t_2(\bar{\sigma}_o - \sigma_1)}{t_1(\bar{\sigma}_o - \sigma_2)} \right] \quad [\text{Eq 33}]$$

After  $n$  is determined,  $B$  can be determined from

$$\left. \begin{aligned} B &= \frac{(\bar{\sigma}_o - \sigma_1)^{n+1}}{(n+1)\bar{\sigma}_o\sigma_1^n t_1} \\ \text{or} \\ B &= \frac{(\bar{\sigma}_o - \sigma_2)^{n+1}}{(n+1)\bar{\sigma}_o\sigma_2^n t_2} \end{aligned} \right\} \quad [\text{Eq 34}]$$

For example, if at  $\sigma_1 = 0.9\sigma_o$  with  $t_1 = 0.484$  and  $\sigma_2 = 0.75\sigma_o$  with  $t_2 = 294$  then one obtains  $n = 5$  and  $B = 3.5 \times 10^{-6}$ .

### **Concrete Scaling**

Scaling of a concrete surface is caused largely by freeze-thaw cycling of the environment. The mechanism causing material damage from freeze-thaw cycling is extremely

complex, involving the moisture content at the time of freezing, the rate of freezing, and the pore structure of the material (which can also alter the freezing temperatures). To develop a simplified model that proves the concept by making qualitatively correct projections—which is the present purpose—it is necessary to make some broad assumptions.

First it is assumed that the number of freeze-thaw cycles is related to the average temperature multiplied by the time over which the temperature was averaged. (Clearly the model will behave best if one takes time increments small enough to sense seasonal changes, but with this model one does not need to take them small enough to resolve daily fluctuations). Next, it is assumed that the surface deteriorates from the outside-in, with the amount of deterioration during a time step being given by the number of freeze-thaw cycles. The depth of scaling  $s(t)$ , shown in Figure 7, is thus modeled as

$$\left. \begin{aligned} \frac{ds}{dt} &= f_s(T, \theta, c) \\ s(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 35}]$$

where the driving function has the form

$$f_s(T, \theta, c) = C < T_o - T(t) > [\theta(t)]^n [1 + c(t)]^m \quad [\text{Eq 36}]$$

and where the parameters  $C$ ,  $n$ , and  $m$  affect the rate of scaling caused by moisture and concentration of chemicals, much like the corrosion model. The notation  $\langle x \rangle$  means that  $\langle x \rangle = x$  if  $x > 0$  and  $\langle x \rangle = 0$  if  $x < 0$ . The value  $T_o$  is the fixed temperature above which no freeze-thaw cycling is expected to occur. For example, if the average temperature over a month is 10 °F one might expect daily fluctuations to result in freeze-thaw cycling. While one might expect less cycling for very cold temperatures, it is assumed for the current problem that the temperature of the material is affected by its ability to absorb the sun's heat. For example, blacktop can thaw even if the air temperature does not exceed the freezing temperature for water.

Assuming the environmental factors to be constant over the time step, Equation 35 can be integrated to give

$$\left. \begin{aligned} s(t_{i+1}) &= s(t_i) + \Delta t f_s(T_a, \theta_a, c_a) \\ s(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 37}]$$

The model parameters can be adjusted to fit anecdotal data. The influence of moisture should decrease rapidly as the moisture level falls below saturation. Thus, an exponent like  $n=5$  might be reasonable. The influence of the chemical concentration might be adequately reflected by a value like  $m=3$ . The value of  $C$  might best be determined by considering the time it would take to scale to a certain depth in the harshest environment.

### Durability Model for Steel

Degradation of the steel beam is caused by corrosion of the exposed surfaces, as shown schematically in Figure 8. The strength of the steel beam at time  $t$  is given by

$$P_u(t) = \frac{f_y}{L} [h^2[t_w - 2x(t)] + 4hb[t_f - 2x(t)]] \quad [\text{Eq 38}]$$

where all of the plate faces are corroded to a depth of  $x(t)$ . The strength of the steel  $f_y$ , the depth of the beam  $h$ , and the width of the beam  $b$  are taken as constants. It is assumed that there are no long-term deflection problems for a steel beam.

#### *Corrosion of Steel*

The steel beam has an I-type cross-section with depth  $h$  and flange width  $b$ . It has a flange thickness  $t_f$  and a web thickness  $t_w$  that, due to the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ , have corroded by amount  $x$  on each face. Figure 9 shows the measure of material loss caused by corrosion.

Corrosion of the steel beam will be modeled in the same manner as the reinforcement bars were modeled for the reinforced concrete beam, but a modification must be made in the environmental exposure. Since steel is almost always painted, a paint model is considered in conjunction with the corrosion model. The moisture and concentration of aggressive chemicals in the corrosion model will simply be the amount transported through the surfacing material.

The following diffusion equation describes the loss of plate thickness caused by corrosion after the  $v$ th repair:

$$\left. \begin{array}{l} x^v(0) = 0 \\ \frac{dx^v}{dt} = \frac{f_c(T, \theta_p, c_p)}{x^v(t)} \\ x^v(t_0^v) = x^{v-1}(t_n^{v-1}) \end{array} \right\} \quad [\text{Eq 39}]$$

where the  $v$ th repair is executed at time  $t_0^v \equiv t_n^{v-1}$  with the initial condition for the next stage being given by the end condition of the previous stage. The cross-section starts in an undamaged condition. The corrosion-driving function is given by the expression

$$f_c(T, \theta_p, c_p) = D_o [\theta_p(t)]^n [1 + c_p(t)]^m e^{-\beta/T(t)} \quad [\text{Eq 40}]$$

where  $\theta_p(t)$  is the moisture content as a percentage of saturation, and  $c_p(t)$  is the concentration of aggressive chemicals transported through the surfacing material.  $T(t)$  is the absolute temperature (Kelvin). The parameters  $n$ ,  $m$ ,  $D_o$ , and  $\beta$  govern the rate of corrosion. These parameters can be established in exactly the same manner as in the corrosion model for reinforcement bars in the concrete beam model.

### **Surface Porosity Model**

The porosity of the surface  $p(t)$  is governed by a damage model that addresses both coating and substrate condition. The rate of change in porosity is assumed to be proportional to the amount of porosity, and is driven by a function of the environmental factors as follows:

$$\left. \begin{array}{l} \frac{dp^v}{dt} = -f_p(T, \theta, c, v) p^v(t) \\ p^v(t_0^v) = g_p(q_v) \\ p^v(0) = 1 \end{array} \right\} \quad [\text{Eq 41}]$$

where the function driving the change in porosity is

$$f_p(T, \theta, c, v) = D_p [T(t)]^{k_v} [1 + \theta(t)]^{n_v} [1 + c(t)]^{m_v} \quad [\text{Eq } 42]$$

and where

$$\tau(t) = \left[ \frac{T(t) - T_o}{T_1} \right]^2 \quad [\text{Eq } 43]$$

is a normalized measure of the temperature. It has been noted that damage to paint generally occurs at a greater rate for very cold temperatures and for very hot temperatures, but slows for intermediate temperatures (Tooke 1980). For this model the reference temperatures  $T_o = 300^\circ\text{K}$  and  $T_1 = 20^\circ\text{K}$  are used. Deterioration speeds up in the presence of moisture and aggressive chemicals, but it will also proceed in a dry, chemically neutral environment.

The quality of the surfacing determines the condition to which the porosity index returns. The function used in this model is simply taken to be

$$g_p(q_v) = (q_v)^{\frac{1}{5}} \quad [\text{Eq } 44]$$

The parameters  $n$ ,  $m$ , and  $k$  are taken to be functions of the quality of the surfacing (i.e., material used, surface preparation, etc.). The user must specify the quality  $q$  of the initial surfacing and of any subsequent repair surfacing. The quality  $q$  will be specified as a number between 0 (poorest quality) and 1 (best quality). Then the parameters  $n$  and  $m$ , at the  $v$ th repair, can be specified in accord with the quality of the repair using the following relationships

$$\left. \begin{array}{l} n_v = 4/q \\ m_v = 3/q \\ k_v = 1/q \end{array} \right\} \quad [\text{Eq } 45]$$

The maintenance strategy then consists of specifying the times and qualities of the resurfacing.

Equation 41 can be numerically integrated to give the following value of the porosity at time  $t_{i+1}^v$  in terms of the porosity at time  $t_i^v$  after the  $v$ th repair:

$$\left. \begin{aligned} p^v(t_{i+1}^v) &= p^v(t_i^v) e^{f_p(T_a, \theta_a, c_a) \Delta t} \\ p^v(t_0^v) &= g_p(q_v) \\ p^v(0) &= 1 \end{aligned} \right\} \quad [\text{Eq 46}]$$

where  $\Delta t = t_{i+1}^v - t_i^v$ , and the environmental factors, as before, are the trapezoidal averages of values at  $t_i^v$  and  $t_{i+1}^v$ .

As the porosity index increases with damage, the resistance of the surfacing material to moisture and chemical transport decreases. The following simple transport model gives the moisture and chemical concentration at the surface of the steel in terms of the environmental moisture and chemical concentration:

$$\left. \begin{aligned} \theta_p(t) &= [1 - p^v(t)] \theta(t) \\ c_p(t) &= [1 - p^v(t)] c(t) \end{aligned} \right\} \quad [\text{Eq 47}]$$

The solution to the rate equation for corrosion can now be obtained by numerical quadrature to give the loss of plate thickness at time  $t_{i+1}^v$  in terms of the loss at time  $t_i^v$  augmented by the corrosion that takes place between those two times:

$$\left. \begin{aligned} x(t_{i+1}^v) &= \sqrt{x^2(t_i^v) + 2\Delta t f_c(T_a, \theta_{pa}, c_{pa})} \\ x^v(0) &= 0 \\ x^v(t_0^v) &= x^{v-1}(t_{i-1}^v) \end{aligned} \right\} \quad [\text{Eq 48}]$$

where  $\theta_{pa}$  and  $c_{pa}$  are evaluated in a manner analogous to  $\theta_p$  and  $c_p$  in Equation 12.

## Durability Model for Wood

Degradation of the wood beam is caused by decay of the exposed surfaces, as shown schematically in Figure 10. The strength of the wood beam at time  $t$  is given by

$$P_u(t) = \frac{2f_u}{3L} [h - 2x(t)]^2 [b - 2x(t)] \quad [\text{Eq 49}]$$

where all of the beam faces are decayed to a depth of  $x(t)$ . The modulus of rupture of the wood  $f_u$ , the depth of the beam  $h$ , and the width of the beam  $b$  are constants.

### Decay of Wood

The wood beam has a rectangular cross-section with depth  $h$  and width  $b$  that, due to the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ , have decayed by the amount  $x$  on each face. Figure 11 shows the measure of material loss caused by rotting.

The following diffusion equation describes the loss of material caused by decay:

$$\left. \begin{aligned} \frac{dx}{dt} &= \frac{f_d(T, \theta)}{x(t)} \\ x(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 50}]$$

The cross-section starts in an undamaged condition. The decay-driving function is given by the expression

$$f_d(T, \theta) = C_d [1 - e^{-T^2(t)}] [\theta(t)]^n \quad [\text{Eq 51}]$$

where

$$T(t) = \left[ \frac{T(t) - T_o}{T_1} \right]^k \quad [\text{Eq 52}]$$

is a normalized measure of the temperature. It has been noted that decay generally occurs only over a certain range of temperatures (American Institute of Timber Construction 1974). For this model the reference temperatures are taken as  $T_o = 300$

$^{\circ}\text{K}$  and  $T_i = 20 \ ^{\circ}\text{K}$ . The deterioration speeds up in the presence of moisture but will not take place in a dry environment.

The parameters  $n$ ,  $m$ , and  $k$  depend on the quality (hardness) of the wood  $Q$ , with  $Q=1$  being the softest wood and  $Q=10$  being the hardest wood. The following values are recommended:

$$\left. \begin{array}{l} n = \sqrt{Q} \\ \\ k = \frac{\sqrt{Q}}{2} \end{array} \right\} \quad [\text{Eq 53}]$$

As before, the solution to the rate can be obtained by numerical quadrature to give the loss of material at time  $t_{i+1}$  in terms of the loss at time  $t_i$  augmented by the decay that takes place between those two times:

$$\left. \begin{array}{l} x(t_{i+1}) = \sqrt{x^2(t_i) + 2\Delta t f_d(T_\alpha, \theta_\alpha)} \\ \\ x(0) = 0 \end{array} \right\} \quad [\text{Eq 54}]$$

where  $\Delta t = t_{i+1} - t_i$  and the environmental functions, as before, are the trapezoidal average of values at  $t_i$  and  $t_{i+1}$ .

### **Creep Deflection of Wood**

Wood is subject to creep deflections caused by long-term loading. To model the long-term creep behavior a Norton exponential creep law (Kachinov 1986) is employed. The law is assumed to apply to the moment-curvature relationship of the beam as follows:

$$\dot{\kappa}(z,t) = \frac{\dot{M}(z,t)}{EI(t)} + BM^m(z,t) \quad [\text{Eq 55}]$$

where  $\dot{\kappa}$  is the rate of change of curvature of the beam,  $M$  is the bending moment, and  $EI$  is the flexural modulus, which may vary with time due to loss of cross-section. The parameters  $B$  and  $m$  are the creep coefficients.

For the model problem  $M(z) = Pz/2$ , enabling the integration of Equation 55 with respect to the spatial dimension  $z$ , and resulting in an equation for the rate of change of deflection as follows:

$$\dot{w}(z,t) = \phi(z,t)\dot{P}(t) + \psi(z)P^m(t) \quad [\text{Eq } 56]$$

where the beam deflected-shape functions are given by

$$\left. \begin{aligned} \phi(z,t) &= \frac{3zL^2 - 4z^3}{48EI(t)} \\ \psi(z) &= B \frac{(m+2)zL^{m+1} - 2^{m+1}z^{m+2}}{(m+1)(m+2)2^{2m+1}} \end{aligned} \right\} \quad [\text{Eq } 57]$$

The maximum deflection occurs at  $z=L/2$ . Any variable denoted with a hat (^) both depends on  $z$  and is evaluated at  $z=L/2$ . Equation 56 can be numerically integrated to give

$$\hat{w}(t_{i+1}) = \hat{w}(t_i) + \hat{\phi}_\alpha [P(t_{i+1}) - P(t_i)] + \hat{\psi} P_\alpha^m \Delta t \quad [\text{Eq } 58]$$

where  $\Delta t = t_{i+1} - t_i$ , and the load and shape functions are a weighted average of the values at  $t_i$  and  $t_{i+1}$  as follows:

$$\left. \begin{aligned} \hat{\phi}_\alpha &= (1-\alpha)\hat{\phi}(t_i) + \alpha\hat{\phi}(t_{i+1}) \\ P_\alpha &= (1-\alpha)P(t_i) + \alpha P(t_{i+1}) \end{aligned} \right\} \quad [\text{Eq } 59]$$

In general, one should use  $\alpha=0.5$ , i.e., the trapezoidal rule.

It is of interest to note that for a constant load  $P$  the beam will creep linearly with time. If it is given that  $m=1$ , and that the beam will creep to 10 times its elastic deflection in 10 years, then the parameter  $B$  is given by  $1/12EI$  ( $\text{ksi}^* \cdot \text{mo.}$ )<sup>1</sup>.

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\* ksi: kips per square inch; 1 kip (kilopound) equals 1,000 lb.

## Durability Model for Aluminum

Degradation of the aluminum beam will be treated the same as for the steel beam except that there will be no surface protection model. Degradation is caused by corrosion of the exposed surfaces, as previously shown schematically in Figure 8. The strength of the aluminum beam at time  $t$  is given by

$$P_u(t) = \frac{f_y}{L} [h^2 t_w - 2x(t)] + 4hb[t_f - 2x(t)] \quad [\text{Eq } 60]$$

where all of the plate faces are corroded to a depth of  $x(t)$ . The strength of the aluminum  $f_y$ , the depth of the beam  $h$ , and the width of the beam  $b$ , are taken as constants. It is assumed that there are no long-term deflection problems for an aluminum beam.

### **Corrosion of Aluminum**

The aluminum beam has an I-type cross-section with depth  $h$  and flange width  $b$ . It has a flange thickness  $t_f$  and a web thickness  $t_w$  that, due to the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ , have corroded by the amount  $x$  on each face. Figure 9, shown previously, illustrates the measure of material loss caused by corrosion. Corrosion of the aluminum will be modeled in exactly the same manner as for steel model, but the corrosion rate is much slower for aluminum.

The following diffusion equation describes the loss of plate thickness caused by corrosion:

$$\left. \begin{aligned} \frac{dx}{dt} &= \frac{f_c(T, \theta, c)}{x^2(t)} \\ x(0) &= 0 \end{aligned} \right\} \quad [\text{Eq } 61]$$

The cross-section starts in an undamaged condition. The corrosion-driving function is given by the expression

$$f_c(T, \theta, c) = D_o[\theta(t)]^n [1+c(t)]^m e^{-\beta/T(t)} \quad [\text{Eq } 62]$$

The parameters  $n$ ,  $m$ ,  $D_o$ , and  $\beta$  govern the rate of corrosion. These parameters can be established in exactly the same manner as in the model for steel corrosion.

The solution to the rate equation for corrosion can be obtained by numerical quadrature to give the loss of plate thickness at time  $t_{i+1}^v$  in terms of the loss at time  $t_i^v$  augmented by the corrosion that takes place between those two times:

$$\left. \begin{aligned} x(t_{i+1}) &= [x^3(t_i) + 3\Delta t f_c(T_a, \theta_a, c_a)]^{1/3} \\ x(0) &= 0 \end{aligned} \right\} \quad [\text{Eq 63}]$$

## Durability Model for FRP

### *Creep Deflection of FRP*

FRP is subject to creep deflections caused by long-term loading. To model the long-term creep behavior a Norton exponential creep law (Kachinov 1986) was adopted and assumed to apply to the moment-curvature relationship of the beam as follows:

$$\dot{\kappa}(z,t) = \frac{M(z,t)}{EI(t)} + BM^m(z,t) \quad [\text{Eq 64}]$$

where  $\dot{\kappa}$  is the rate of change of curvature of the beam,  $M$  is the bending moment, and  $EI$  is the flexural modulus (the latter of which may vary with time due to loss of cross-section). The parameters  $B$  and  $m$  are the creep coefficients.

For the model problem  $M(z) = Pz/2$ , enabling the integration of Equation 64 with respect to the spatial dimension  $z$ , and resulting in an equation for the rate of change of deflection as follows:

$$\dot{w}(z,t) = \phi(z,t)\dot{P}(t) + \psi(z)P^m(t) \quad [\text{Eq 65}]$$

where the beam deflected-shape functions are given by

$$\left. \begin{aligned} \phi(z,t) &\equiv \frac{3zL^2 - 4z^3}{48EI(t)} \\ \psi(z) &\equiv B \frac{(m+2)zL^{m+1} - 2^{m+1}z^{m+2}}{(m+1)(m+2)2^{2m+1}} \end{aligned} \right\} \quad [\text{Eq 66}]$$

The maximum deflection occurs at  $z=L/2$ . A hat is used to denote any variable that both depends on  $z$  and is evaluated at  $z=L/2$ . Equation 65 can be numerically integrated to give

$$\hat{w}(t_{i+1}) = \hat{w}(t_i) + \hat{\phi}_\alpha [P(t_{i+1}) - P(t_i)] + \hat{\psi} P_\alpha^m \Delta t \quad [\text{Eq 67}]$$

where  $\Delta t = t_{i+1} - t_i$ , and the load and shape functions are a weighted average of the values at  $t_i$  and  $t_{i+1}$  as follows:

$$\left. \begin{aligned} \hat{\phi}_\alpha &= (1-\alpha)\hat{\phi}(t_i) + \alpha\hat{\phi}(t_{i+1}) \\ P_\alpha &= (1-\alpha)P(t_i) + \alpha P(t_{i+1}) \end{aligned} \right\} \quad [\text{Eq 68}]$$

In general, one should use  $\alpha=0.5$ , i.e., the trapezoidal rule.

It is of interest to note that for a constant load  $P$  the beam will creep linearly with time. If it is given that  $m=1$  and that the beam will creep to 10 times its elastic deflection in 10 years, then the parameter  $B$  is given by  $1/12EI$  ( $ksi\cdot mo.$ )<sup>-1</sup>.

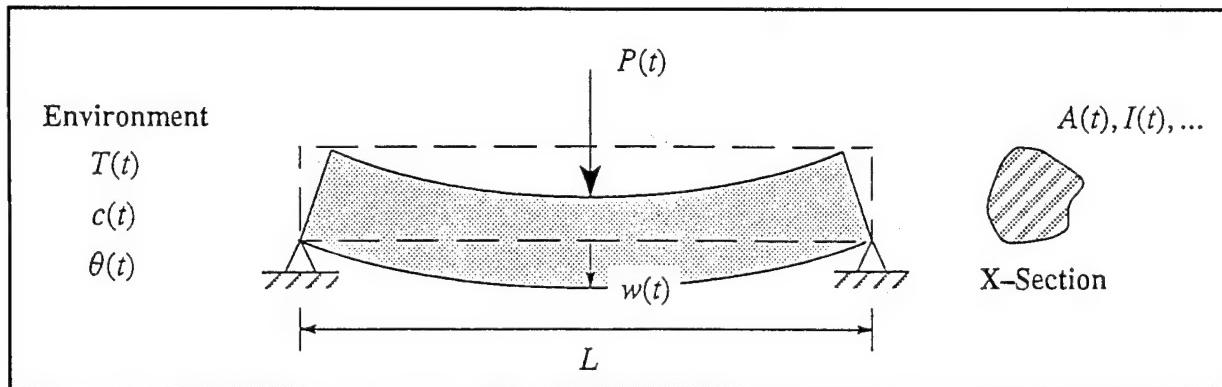


Figure 4. The model problem for durability of a structural element.

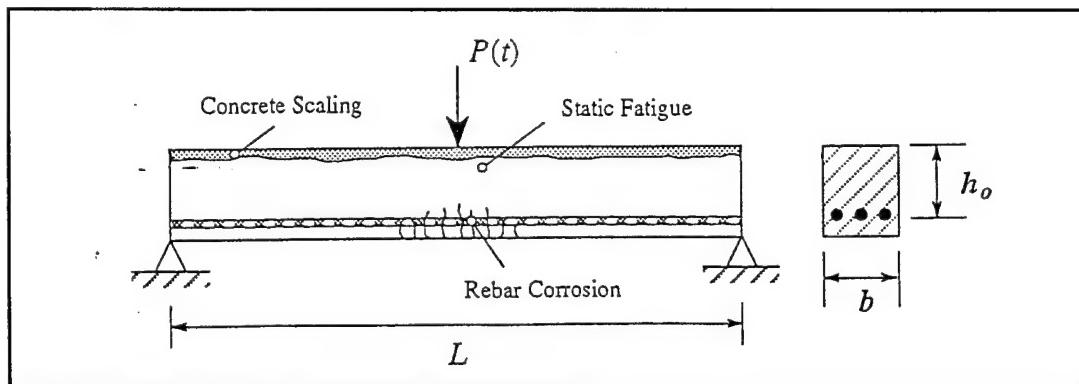


Figure 5. The model problem for reinforced concrete.

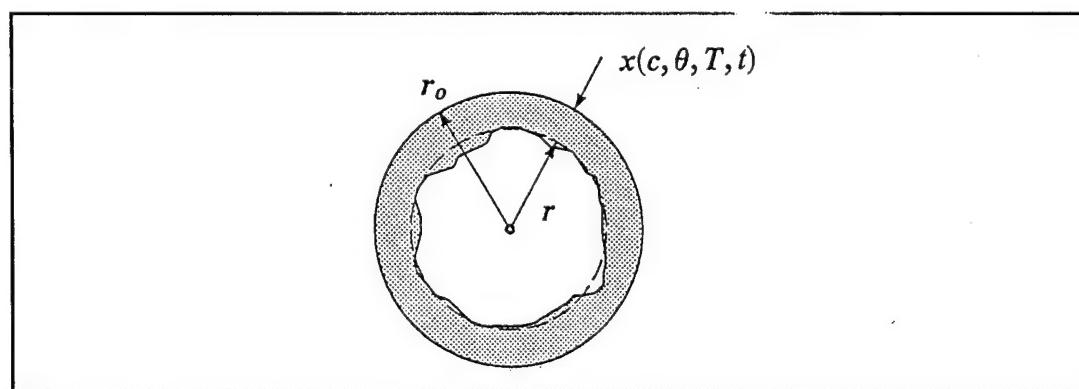


Figure 6. Measure of reinforcement bar corrosion.

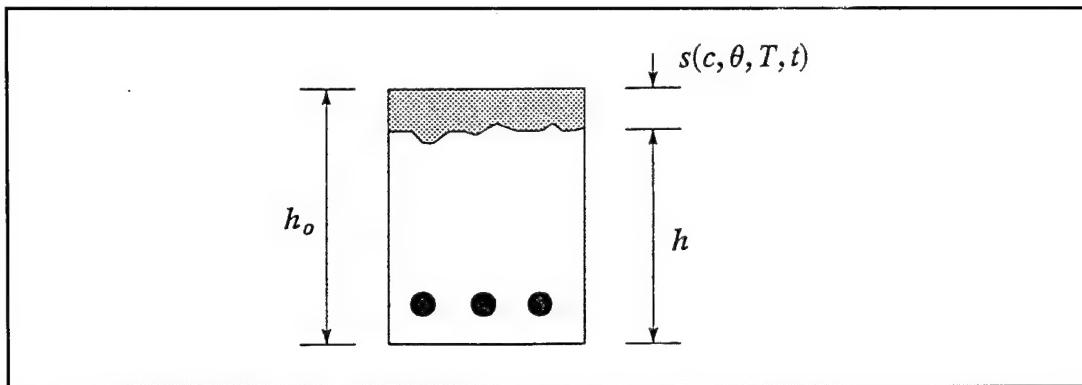


Figure 7. Measure of concrete scaling depth.

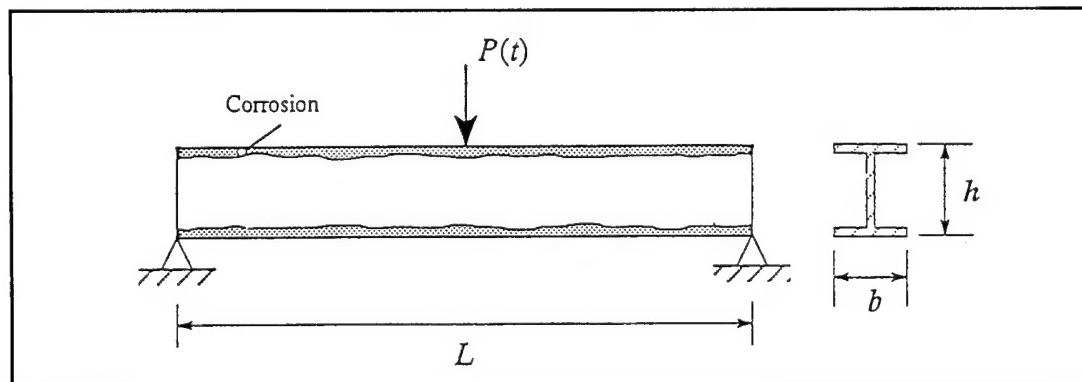


Figure 8. The model problem for steel.

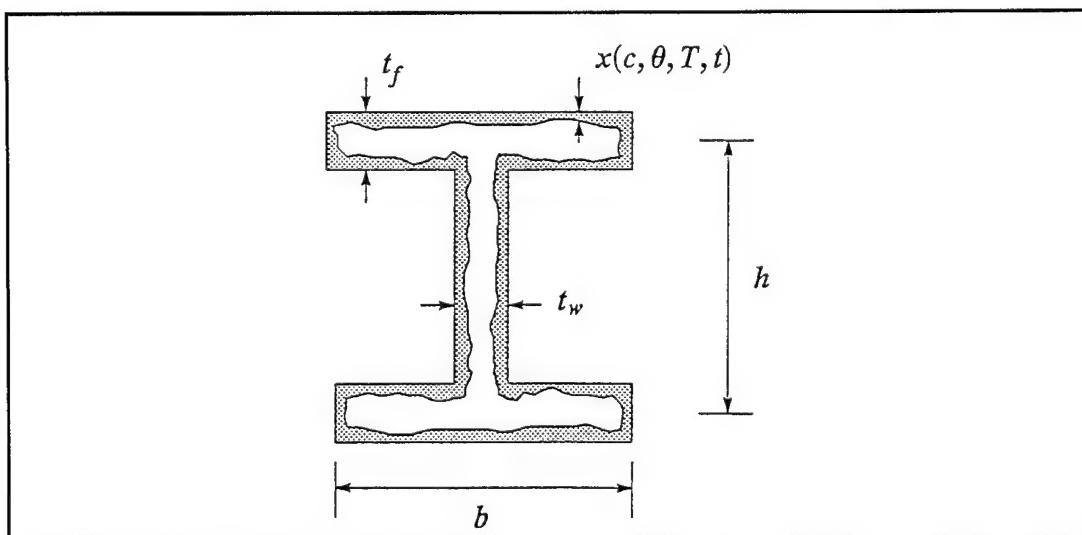


Figure 9. Measure of steel corrosion depth.

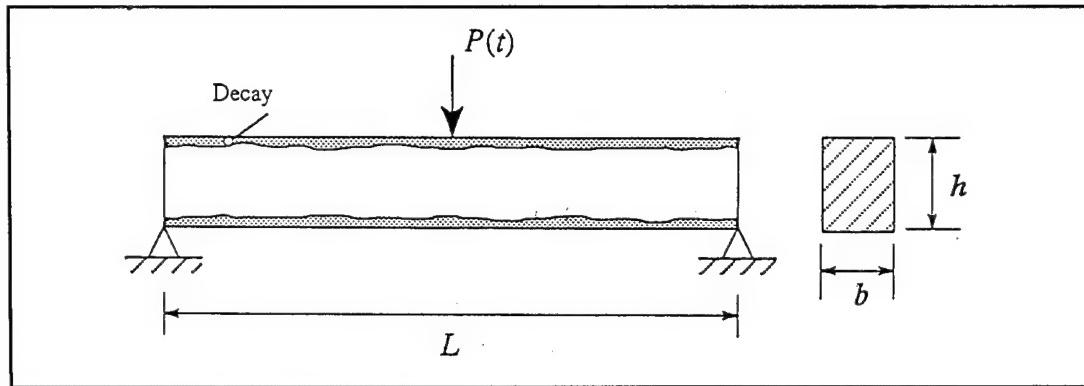


Figure 10. The model problem for wood.

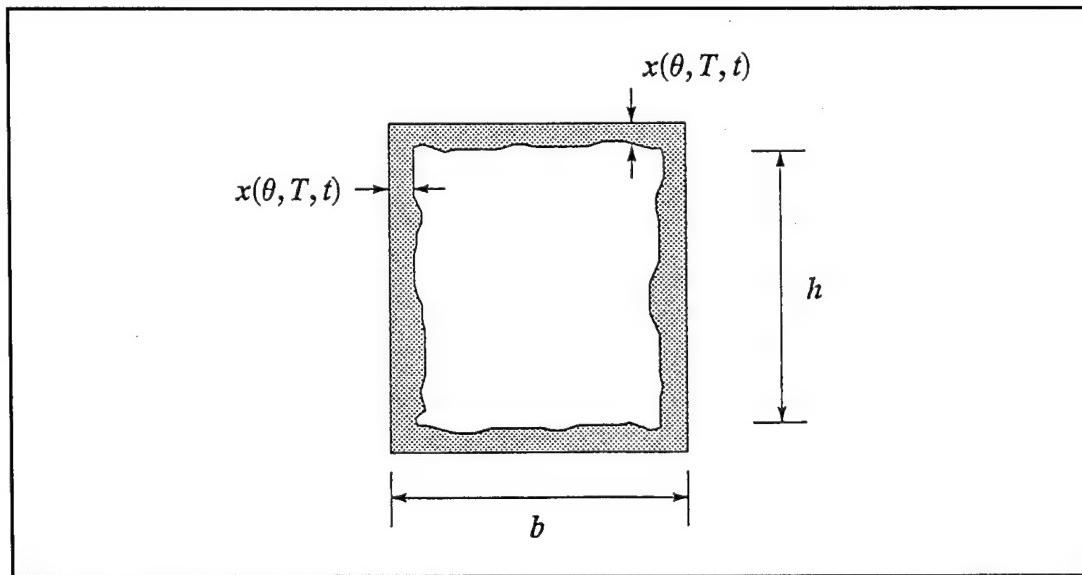


Figure 11. Measure of wood decay depth.

## 4 Modeling the Durability of a Cladding Element

### Description of Model Problem

As a model problem consider the durability of a cladding element of unit area exposed to the environment. The surface material can be paint, brick, aluminum, polyvinyl-chloride (PVC), or EIFS (exterior insulation and finish system). The surface is subjected to environmental factors that fluctuate over time. Important among these factors for this model problem are:

- ambient temperature  $T(t)$
- ambient moisture  $\theta(t)$  due to humidity and precipitation, expressed as a fraction of saturation
- concentration of aggressive chemicals or agents of decay  $c(t)$ , expressed as a fraction of the maximum concentration.

### Serviceability Criterion

The main concern for the example cladding systems is serviceability, which includes both aesthetic appeal and protection quality. The serviceability may degrade over time because of corrosion, scaling, cracking, peeling, or other causes. Degradation of serviceability can be modeled with rate equations much as was done for the structural element (see Chapter 3). In each case, the serviceability will be indexed with a single parameter  $a(t)$ . The initial serviceability index will be *unity*, and will degrade from there. It is assumed that the cladding element is serviceable if

$$a(t) \geq \gamma \quad [\text{Eq 69}]$$

where  $\gamma$  is the *serviceability limit*. One can use this limit to make repair decisions (i.e., repair if  $a(t) < \gamma$ ).

## Durability Model for Paint

The serviceability of the painted surface  $a(t)$  is governed by a damage model. It is assumed that the rate of change of the serviceability index is inversely proportional to the value of the index, and is driven by a function of the environmental factors as follows:

$$\left. \begin{aligned} \frac{da^v}{dt} &= -\frac{f_p(T, \theta, c, v)}{a^v(t)} \\ a^v(t_0) &= g_p(q_v) \\ a^v(0) &= 1 \end{aligned} \right\} \quad [\text{Eq } 70]$$

where the function driving the change in serviceability is

$$f_p(T, \theta, c, v) = D_p[\tau(t)]^\alpha [1 + \beta\theta(t)][1 + \gamma c(t)] \quad [\text{Eq } 71]$$

and where

$$\tau(t) = \left[ \frac{T_0 - T(t)}{T_0} \right]^2 \quad [\text{Eq } 72]$$

is a normalized measure of the temperature. It has been noted that damage to paint generally occurs at a greater rate for very cold temperatures and for very hot temperatures, but slows for intermediate temperatures. Here the reference temperature is taken as  $T_0 = 60F$ . Deterioration speeds up in the presence of moisture and aggressive chemicals, but proceeds even in a dry, chemically neutral environment. The constant  $D_p$  is taken to be  $0.015 \text{ sec}^{-1}$ .

The quality of the surfacing determines the condition to which the porosity index returns. The function used in this model is simply taken to be

$$g_p(q) = 0.9 + 0.1q \quad [\text{Eq } 73]$$

The parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  and are taken to be functions of the quality of the surfacing (i.e., material used, surface preparation, etc.). The user must specify the quality  $q$  of the initial surfacing and of any subsequent repair surfacing. The quality  $q$  will be specified as a number between 0 (poorest quality) and 1 (best quality). Then the parameters at the  $v$ th repair can be specified in accord with the quality of the repair using the following relationships:

$$\left. \begin{array}{l} \beta_v = 1 - q_v \\ \gamma_v = 1 - q_v \\ \alpha_v = 5q_v \end{array} \right\} \quad [\text{Eq 74}]$$

The maintenance strategy then consists of specifying the times and qualities of the resurfacing. For current purposes it is assumed that the repair program consists of a single quality of repair executed each time the serviceability index inequality is violated.

Equation 70 can be numerically integrated to give the following value of the porosity at time  $t_{i+1}$  in terms of the porosity at time  $t_i$  after the  $v$ th repair:

$$\left. \begin{array}{l} a^v(t_{i+1}) = \sqrt{[a^v(t_i)]^2 - 2f_p(T_\alpha, \theta_\alpha, c_\alpha)\Delta t} \\ a^v(t_0) = g_p(q) \\ a^0(0) = 1 \end{array} \right\} \quad [\text{Eq 75}]$$

where  $\Delta t = t_{i+1}^v - t_i^v$ , and the environmental factors are the trapezoidal averages of values at  $t_i^v$  and  $t_{i+1}^v$  as before.

## Durability Model for EIFS

EIFS is similar to paint in two important respects: it is used as an external cladding and it is a polymeric material. Therefore, for the current work it is assumed that the

serviceability of EIFS is essentially the same as paint, but with different constants used in the degradation model. The constant  $D_p$  is taken to be  $0.002 \text{ sec}^{-1}$ .

### Durability Model for Aluminum

Degradation of the aluminum surface will be caused by corrosion, an oxidation process the rate of which increases exponentially with temperature. The aluminum surface quality  $a(t)$  decays over time due to the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ . The following equation describes the loss of serviceability caused by corrosion:

$$\left. \begin{aligned} \frac{da}{dt} &= -f_c(T, \theta, c)a(t) \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq 76}]$$

The surface starts in an undamaged condition. The corrosion-driving function is given by the expression

$$f_c(T, \theta, c) = A[\theta(t)]^n[1 + c(t)]^m e^{-\beta/T(t)} \quad [\text{Eq 77}]$$

The parameters  $n$ ,  $m$ ,  $A$ , and  $\beta$  govern the rate of corrosion. These parameters can be established in exactly the same manner as for the steel corrosion model. The values used in this case were  $n=1$ ,  $m=3$ ,  $A=1.5 \times 10^{-9} \text{ sec}^{-1}$ , and  $\beta=450K$ .

The solution to the rate equation for corrosion can be obtained by numerical quadrature to give the serviceability at time  $t_{i+1}$ :

$$\left. \begin{aligned} a(t_{i+1}) &= a(t_i)e^{-\int f_c(T_a, \theta_a, c_a) \Delta t} \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq 78}]$$

## Durability Model for PVC

Degradation of the PVC surface will be caused by ultraviolet radiation from sunlight. To simplify this discussion it is assumed that ultraviolet exposure is a function of ambient temperature. The surface deteriorates by an amount  $a(t)$  due to the history of temperature  $T$  and chemical exposure  $c$ . PVC is impervious to moisture. The following diffusion equation describes the loss of serviceability due to ultraviolet degradation:

$$\left. \begin{aligned} \frac{da}{dt} &= -\frac{f_u(T,c)}{a(t)} \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq 79}]$$

The surface starts in an undamaged condition. The driving function is given by the expression

$$f_u(T,\theta,c) = A e^{\tau(t)} [1 + c(t)]^m \quad [\text{Eq 80}]$$

where

$$\tau(t) \equiv \frac{T(t) - T_o}{T_o} \quad [\text{Eq 81}]$$

is a normalized measure of the temperature. Damage caused by ultraviolet exposure generally occurs only at higher temperatures (i.e., when exposure to sunlight is more intensive). Here the reference temperature is taken as  $T_o = 300 \text{ } ^\circ\text{K}$ . Deterioration also speeds up in the presence of chemicals. The parameters  $m=1$  and  $A = 0.5 \times 10^{-9} \text{ sec}^{-1}$  govern the rate of degradation.

The solution to the rate equation can be obtained by numerical quadrature to give the serviceability at time  $t_{i+1}^v$ :

$$\left. \begin{aligned} a(t_{i+1}) &= \sqrt{a^2(t_i) - 2\Delta t f_u(T_\alpha, c_\alpha)} \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq } 82]$$

### Durability Model for Brick

Degradation of the brick surface will take the form of scaling caused by the history of temperature  $T$ , moisture  $\theta$ , and corroding chemicals  $c$ . The following diffusion equation describes the loss of serviceability due to scaling:

$$\left. \begin{aligned} \frac{da}{dt} &= -f_s(T, \theta, c) \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq } 83]$$

The surface starts in an undamaged condition. The driving function is given by the expression

$$f_s(T, \theta, c) = C <T_o - T(t)> [\theta(t)]^n [1 + c(t)]^m \quad [\text{Eq } 84]$$

where the parameters  $C$ ,  $n$ , and  $m$  affect the rate of scaling caused by moisture and concentration of chemicals, much as in the corrosion model. The notation  $<x>$  means that  $<x> = x$  if  $x > 0$  and  $<x> = 0$  if  $x < 0$ . The value  $T_o = 303$  °K is the fixed temperature above which no freeze-thaw cycling is expected to occur. The values used here are  $C = 1.0 \times 10^{-4}$  sec<sup>-1</sup>K<sup>1</sup>,  $n = 3$ , and  $m = 3$ .

The solution to the rate equation can be obtained by numerical quadrature to give the serviceability at time  $t_{i+1}^v$ :

$$\left. \begin{aligned} a(t_{i+1}) &= a(t_i) - 2\Delta t f_c(T_\alpha, \theta_\alpha, c_\alpha) \\ a(0) &= 1 \end{aligned} \right\} \quad [\text{Eq } 85]$$

## 5 Durability Model Test Cases

### Test Case Overview

This chapter presents durability cost results of two BMDM test cases. Two types of building components were tested: a simply-supported beam and a cladding element. The beam materials tested were reinforced concrete and steel; the cladding materials tested were paint and aluminum. The design criteria and durability models for beams and cladding were detailed in Chapters 3 and 4, respectively.

The material systems were tested in two different environments given the generic form described previously in terms of temperature  $T(t)$ , moisture content  $\theta(t)$ , and chemical concentration  $c(t)$ :

$$T(t) = T_o + \Delta T \sin(\pi t/6) + \delta T \eta(t)$$

$$\theta(t) = \theta_o + \Delta \theta \sin(\pi t/6) + \delta \theta \eta(t)$$

$$c(t) = c_o + \delta c \eta(t)$$

The two environments were given the descriptive names "Tropical" and "Continental U.S.," defined by the following parameters and values:

#### *Tropical*

$$\begin{aligned} T_o &= 85^{\circ}\text{F} & \Delta T &= 10^{\circ}\text{F} & \delta T &= 5^{\circ}\text{F} \\ \theta_o &= 90\% & \Delta \theta &= 5\% & \delta \theta &= 1\% \\ c_o &= 0.2 & \delta c &= 0.1 \end{aligned}$$

#### *Continental U.S.*

$$\begin{aligned} T_o &= 40^{\circ}\text{F} & \Delta T &= 50^{\circ}\text{F} & \delta T &= 10^{\circ}\text{F} \\ \theta_o &= 50\% & \Delta \theta &= 30\% & \delta \theta &= 5\% \\ c_o &= 0.7 & \delta c &= 0.3 \end{aligned}$$

Figures 12 and 13 show typical variations of these environmental stimuli over a 120-month period.

### Results for Simply Supported Beam

The reinforced concrete beam degrades over time through three mechanisms: reinforcement bar (rebar), concrete scaling, and static fatigue. The beam is repaired by restoring the member to its original state. The steel beam degrades through corrosion only; a layer of paint is applied to the beam to slow down the corrosion process; but this paint also degrades over time. The steel beam is repainted to a user-specified quality whenever the paint (or surface) quality falls below a certain value. The steel beam itself is repaired only by replacement.

Figures 14 and 15 show some typical results obtained from the BMDM spreadsheet analysis of the reinforced concrete beam subjected to both the tropical and continental U.S. stimuli, respectively. It is apparent in this model that corrosion of the rebar is the only mechanism of deterioration present in the tropical environment, but both rebar corrosion and concrete scaling occur in the continental U.S. environment. Table 1 shows durability costs for the beam, measured by taking averages of 10 simulations. Different time periods (50 and 10 years) and discount rates (5% and 20% annually) are considered for each environment. Over a 10-year period, no repairs are necessary in either environment, so durability cost is the same (i.e., equal to the initial cost). Over 50 years, one repair is needed in both environments. However, because the beam subjected to the continental U.S. environment needs to be repaired sooner (see Figures 14 and 15), the durability cost for the material is also higher. Note in Table 1 that the higher discount rate reduces the cost difference between the two environments because the present value of money at any point in the future decreases as the discount rate increases.

Figures 16 through 19 show sample calculations for the steel beam. Figures 16 and 17 show data obtained using the tropical stimuli, with two maintenance (i.e., repainting) strategies: low quality (0.2, Figure 16) and high quality (0.6, Figure 17).

**Table 1. Durability costs for reinforced concrete beam (averages from 10 BMDM simulations).**

	Tropical		Continental U.S.	
	50 yrs	10 yrs	50 yrs	10 yrs
Cost (\$/m)	50 yrs	10 yrs	50 yrs	10 yrs
5% discount rate	192.57	168.44	205.71	168.44
20% discount rate	168.51	168.44	168.85	168.44

Figures 18 and 19 show similar data for the continental U.S. environment. Note that the repair trigger (i.e., the surface quality measure that triggers repainting) is set at 0.6 in all cases. Table 2 shows average durability costs for the steel beam in the two environments for different maintenance strategies, time periods, and discount rates. In the tropical environment, there is little difference between the costs associated with a high- and low-quality maintenance strategy. The low-quality approach results in one repainting near the end of the 50 year time period, which is inexpensive. However, considerably more painting is required in the continental U.S. if a low-quality maintenance strategy is adopted instead of high-quality maintenance. This results in durability costs two to four times higher than those for a high-quality maintenance strategy in this environment.

## Results for Cladding

Paint and aluminum cladding materials were analyzed by a method similar to that used for the concrete and steel beams. As discussed in Chapter 4, both types of cladding materials suffer degradation of surface quality as measured using a variable between 0 and 1. Paint is maintained by repainting to a specified quality, and the aluminum is repaired by replacement.

Figures 20 through 23 show the behavior of the paint cladding in the two test environments when two repair strategies are used (low-quality, 0.2, and high-quality, 0.8). Table 3 shows average values of the durability costs associated with these strategies for 50- and 10-year periods, using discount rates of 5% and 20%. The repair trigger was held fixed at 0.6 in all cases. In the tropical environment, low-quality repairs are more expensive than high-quality maintenance because no further repainting is required in the latter case. The opposite is true in the continental U.S.; in terms of durability cost the relatively high frequency of low-quality repairs is offset by the low per-repair cost as compared to high-quality (and high-cost) repairs which are also needed frequently. Thus, according to BMDM as run for the continental U.S. environment, frequent cheap repairs would be preferred to fewer, more expensive repaintings.

The behavior and durability costs associated with the aluminum cladding are shown in Figures 24 and 25, and in Table 4. The repair trigger in this case also is fixed at 0.6. Two repairs are required in a 50-year period in the continental U.S. environment, compared to one repair required in the tropics. Therefore, the durability cost is higher in the continental U.S. over the 50-year period. The durability costs are equal to the initial cost over a 10-year period in all cases.

**Table 2. Durability costs for steel beam (averages from 10 BMDM simulations).**

	Tropical				Continental U.S.			
	50 yrs		10 yrs		50 yrs		10 yrs	
Cost (\$/m)	0.2 main	0.6 main	0.2 main	0.6 main	0.2 main	0.6 main	0.2 main	0.6 main
5% discount rate	50.35	49.53	49.53	49.53	206.23	50.26	115.79	49.53
20% discount rate	49.54	49.53	49.53	49.53	90.24	49.54	84.41	49.53

**Table 3. Durability costs for paint cladding (averages from 10 BMDM simulations).**

	Tropical				Continental U.S.			
	50 yrs		10 yrs		50 yrs		10 yrs	
Cost (\$/m)	0.2 main	0.8 main	0.2 main	0.8 main	0.2 main	0.8 main	0.2 main	0.8 main
5% discount rate	18.57	10.00	12.91	10.00	29.97	64.62	17.66	32.78
20% discount rate	11.52	10.00	11.17	10.00	14.50	21.98	13.68	19.99

**Table 4. Durability costs for aluminum cladding (averages from 10 BMDM simulations).**

	Tropical		Continental U.S.	
Cost (\$/m)	50 yrs	10 yrs	50 yrs	10 yrs
5% discount rate	71.28	50.00	97.4	50.00
20% discount rate	50.22	50.00	51.57	50.00

## Discussion

In general, different material behavior and durability costs were observed for the two test environments. The concrete beam simulations demonstrated that the nature of the damage can depend on the environmental stimuli. Concrete scaling was observed only when the beam was subjected to the continental U.S. stimuli, but rebar corrosion was present in both cases.

It was also observed that the selection of the most economical maintenance strategy can depend on the environment. The BMDM simulations suggest high-quality maintenance of the steel beam for the continental U.S. stimuli, but on the other hand, the choice of maintenance strategy in the tropical environment does not produce any noticeable cost differences. According to BMDM simulations for paint cladding, high-quality maintenance is appropriate in the tropics, but low-quality repainting is the most economical maintenance strategy for the continental U.S. environment.

The simulations and observations at this point in the research confirm that BMDM can qualitatively predict and compare the durability cost of various building materials. The algorithms upon which BMDM is based—formulas representing materials durability and environmental stimuli—would need to be extended and refined before BMDM can quantitatively model complex materials degradation and environmental processes. However, the findings of this study clearly indicate that BMDM could be developed into a finished tool to help facility designers and engineers make better decisions in selecting building materials for optimal life-cycle economy and serviceability.

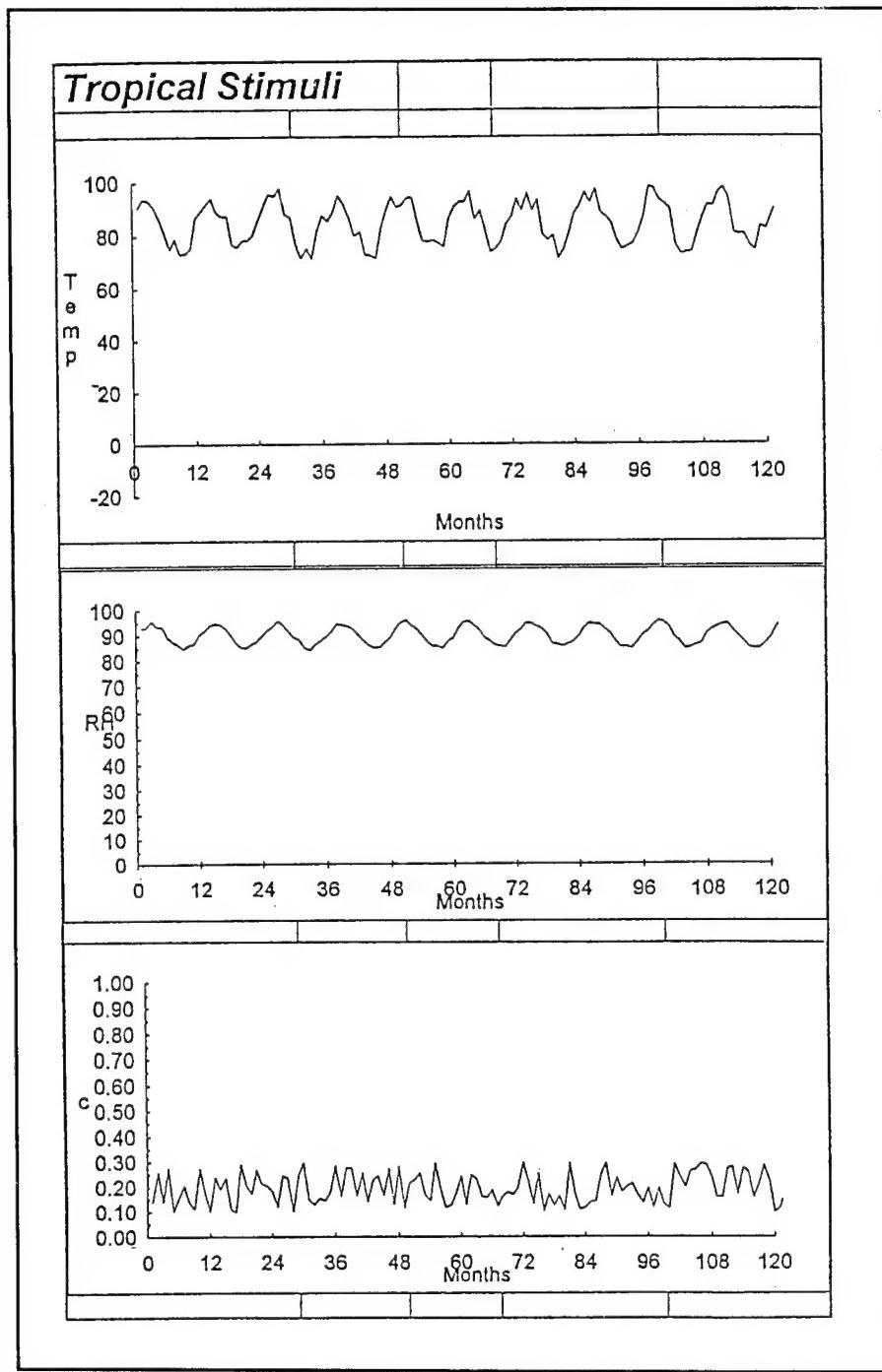


Figure 12. Typical tropical environmental stimuli.

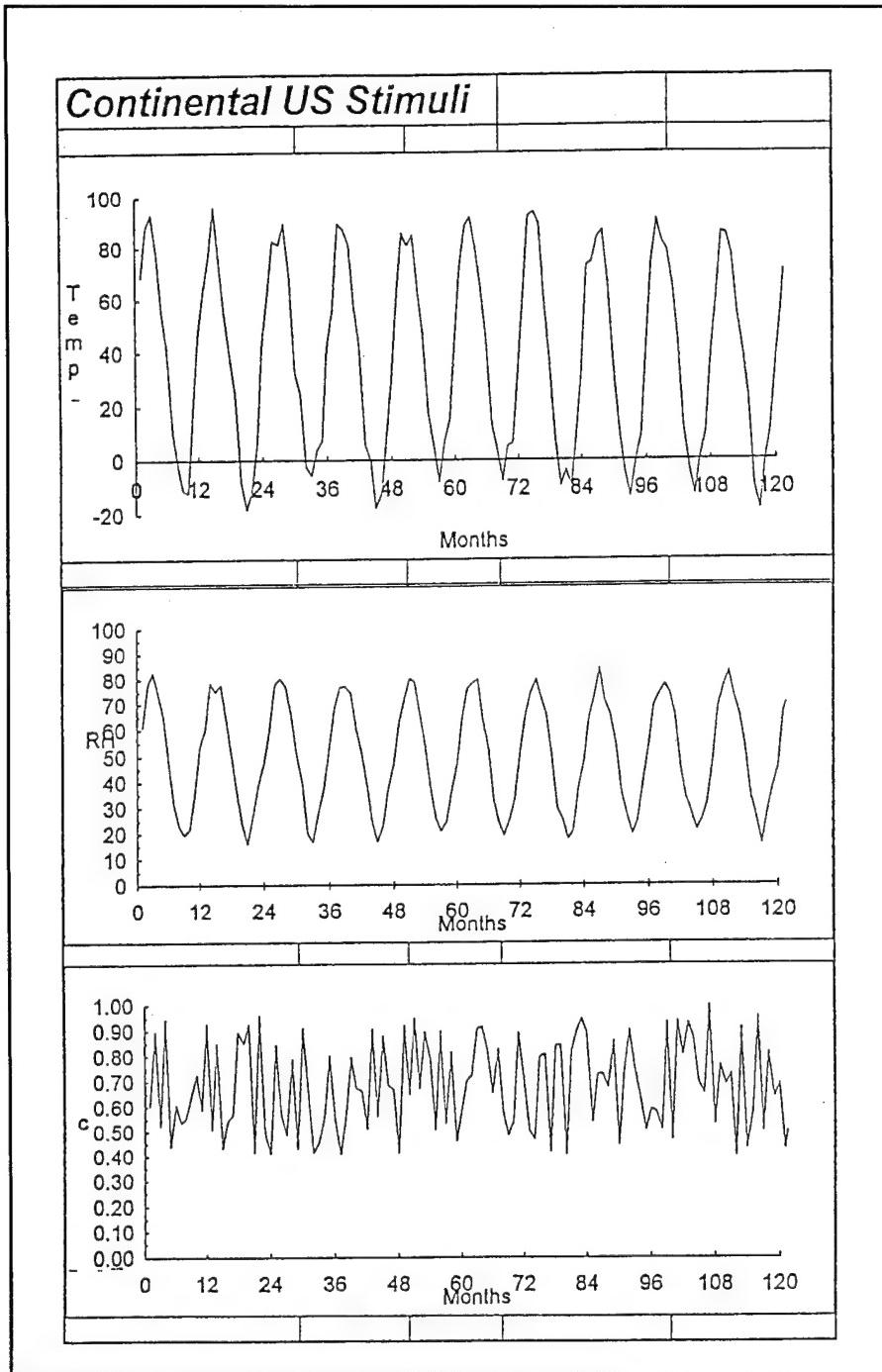


Figure 13. Typical continental U.S. environmental stimuli.

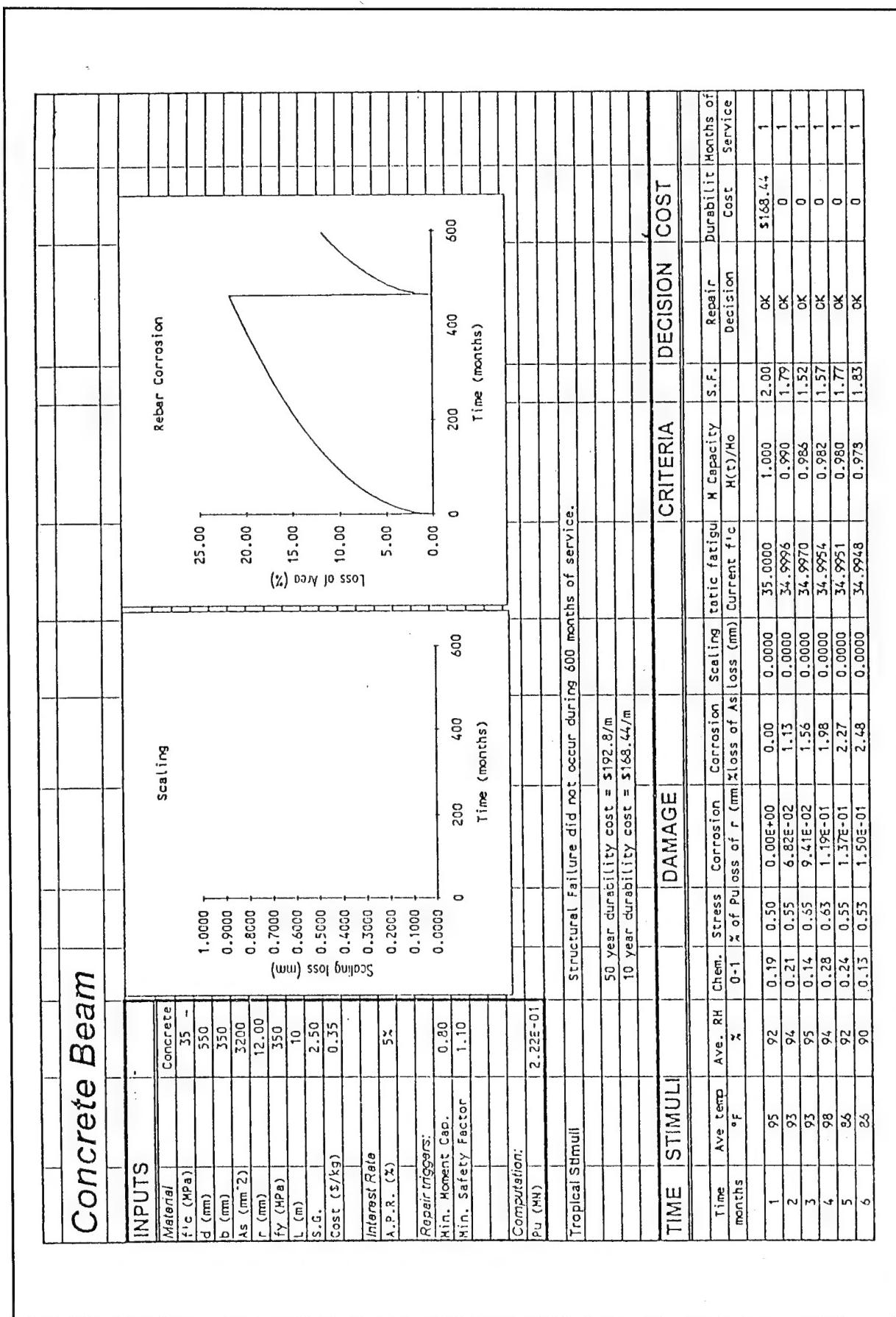


Figure 14. Behavior of concrete beam subjected to tropical environmental stimuli.

## Concrete Beam

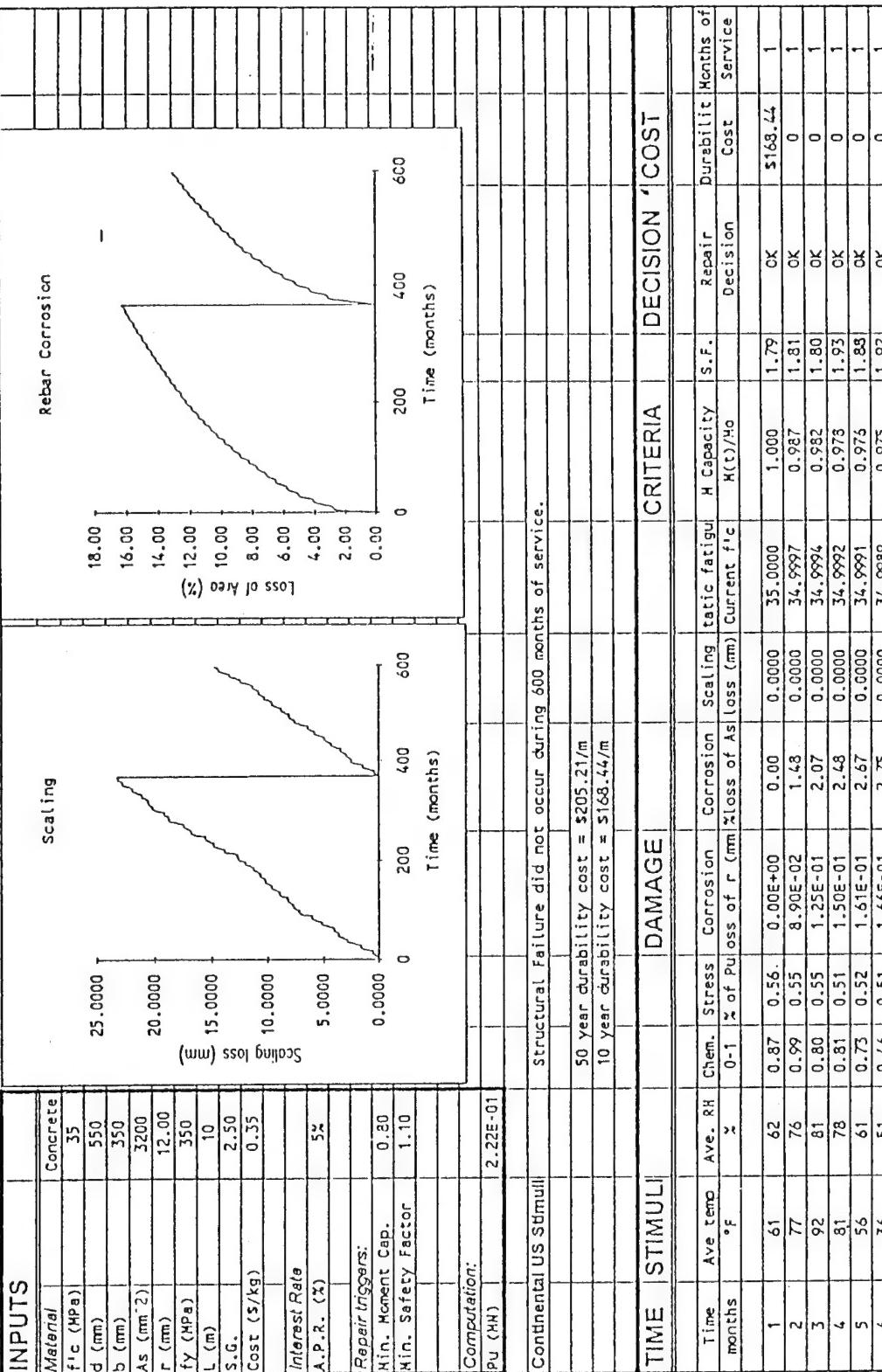


Figure 15. Behavior of concrete beam subjected to continental U.S. environmental stimuli.

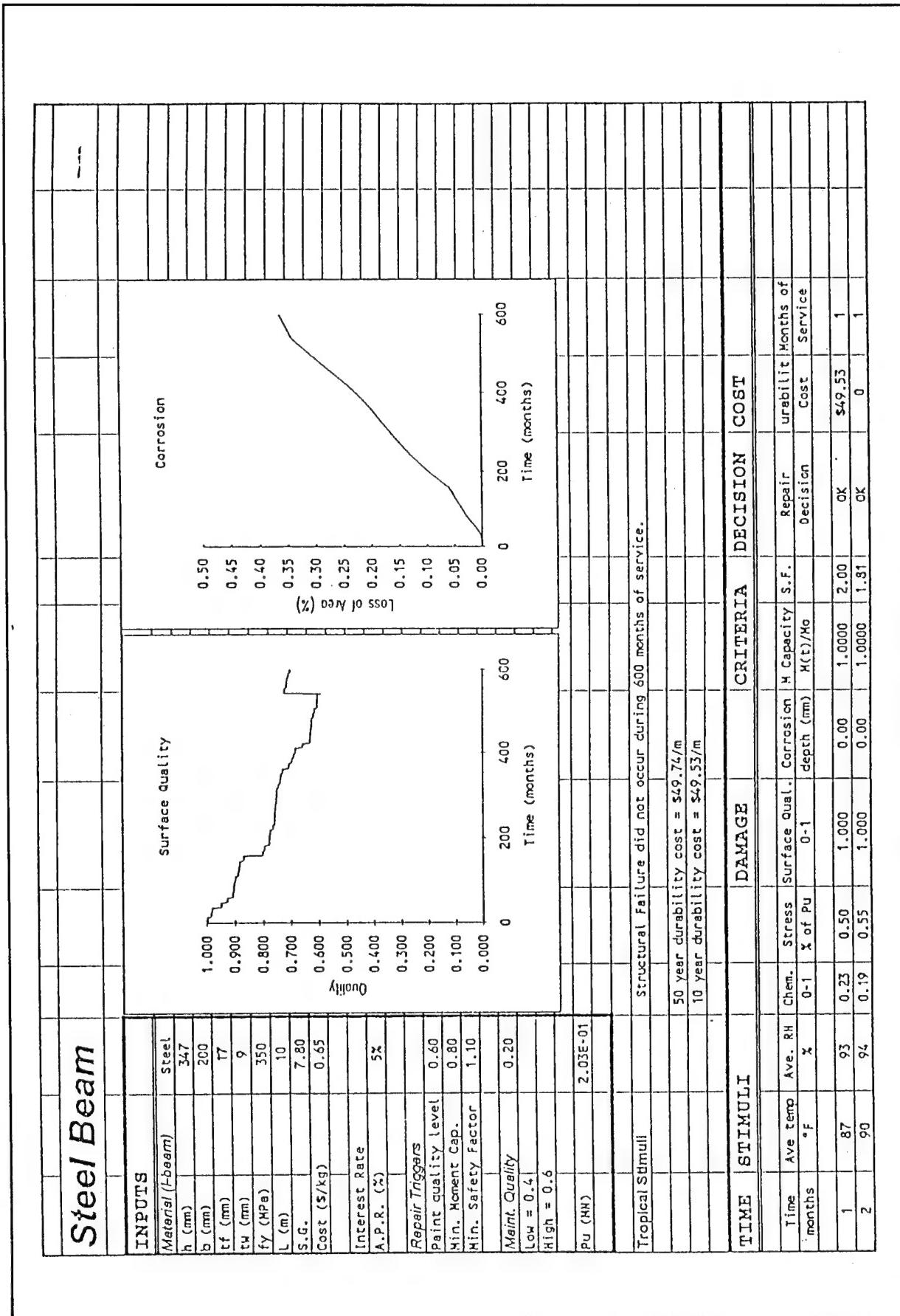
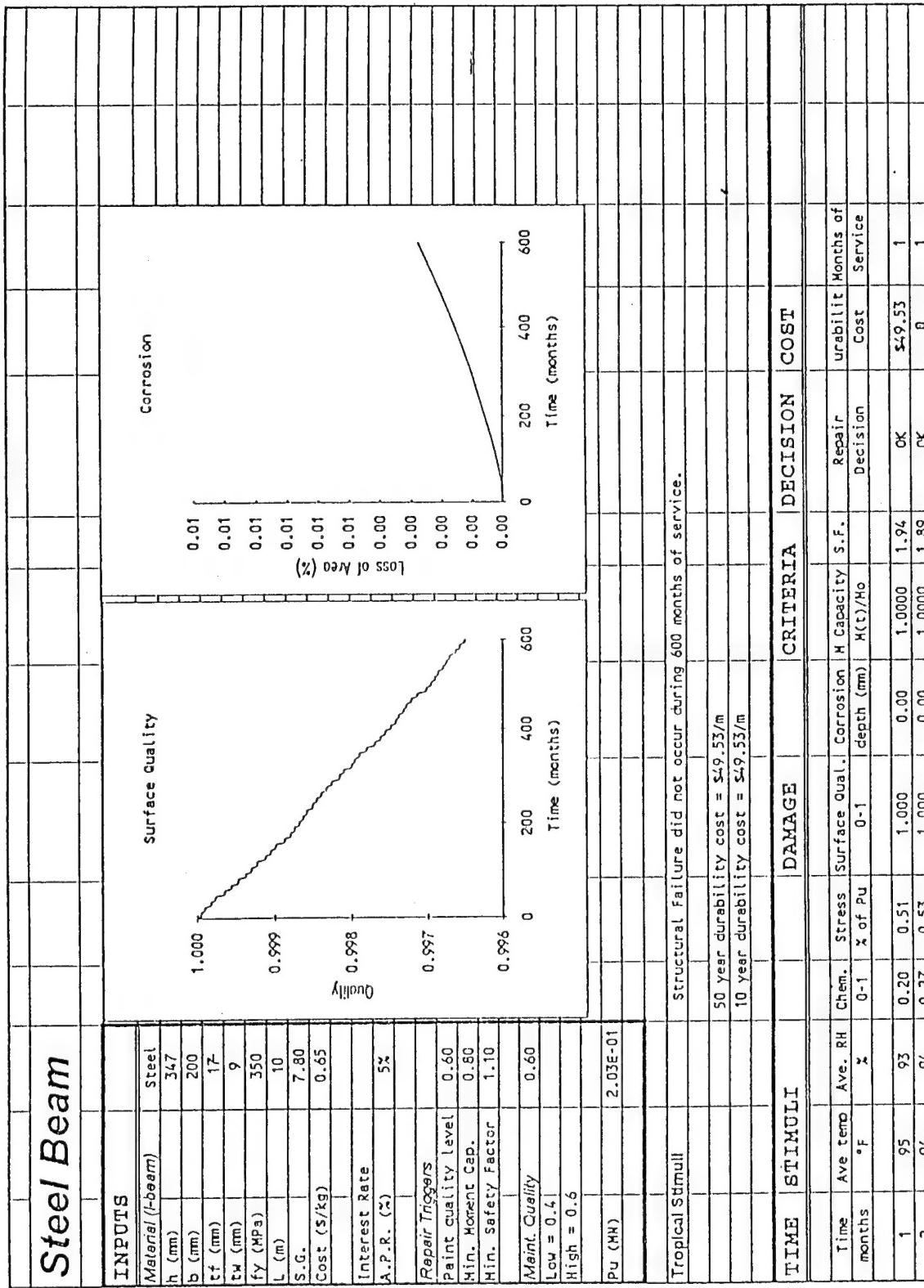


Figure 16. Behavior of steel beam subjected to tropical stimuli (low-maintenance strategy).



**Figure 17.** Behavior of steel beam subjected to tropical stimuli (high-maintenance strategy).

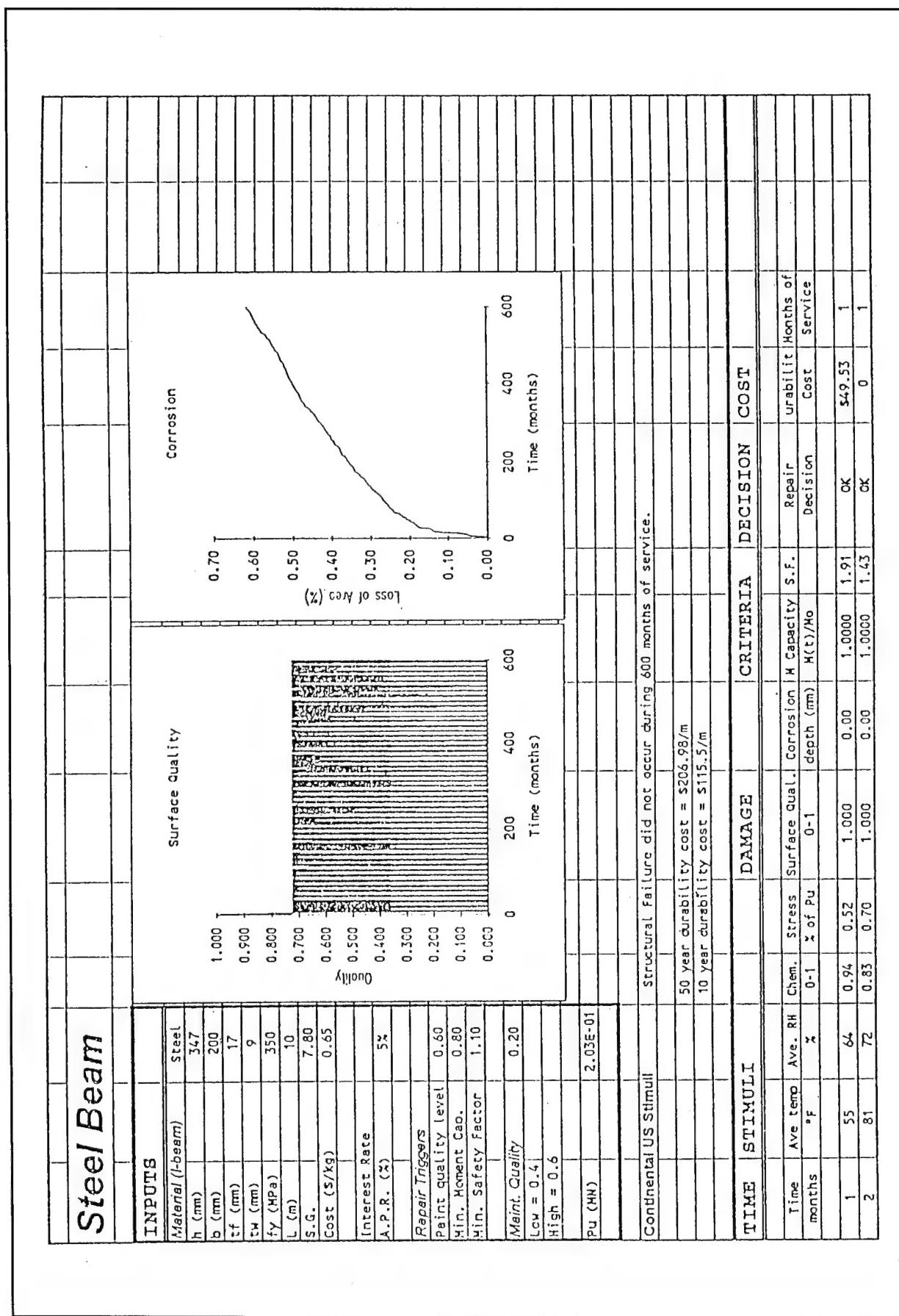


Figure 18. Behavior of steel beam subjected to continental U.S. stimuli (low-maintenance strategy).

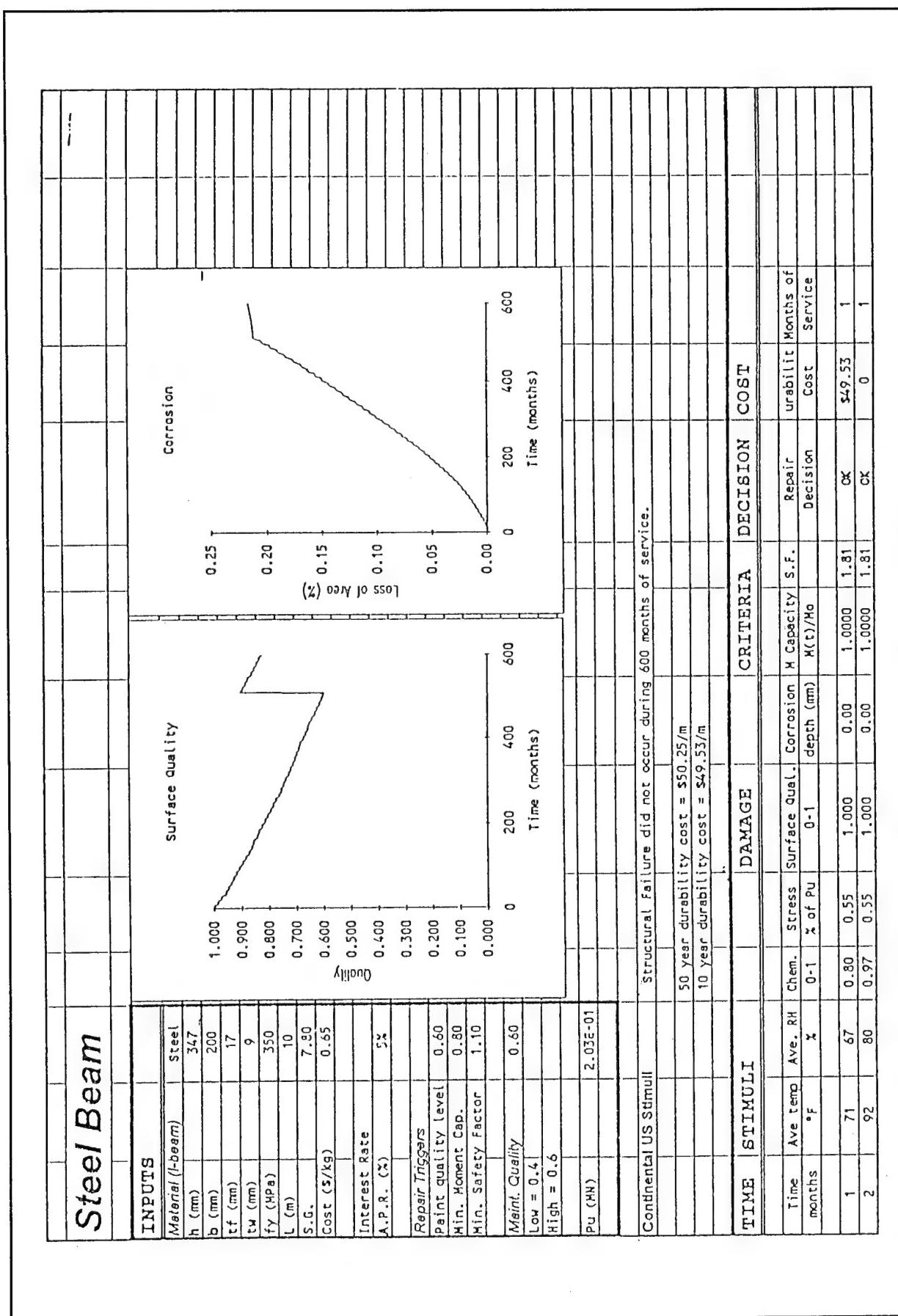


Figure 19. Behavior of the steel beam subjected to continental U.S. stimuli (high-maintenance strategy).

## Paint Cladding

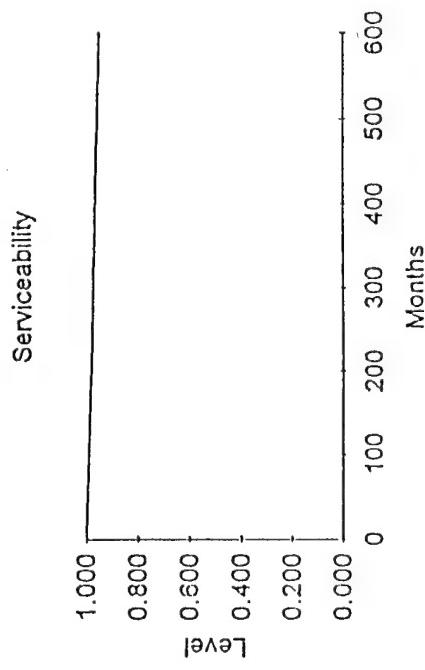
INPUTS		Serviceability	
Material	paint		
Repair trigger	0.60		
Aesthetic level which triggers a repair			
Material/Maintenance	0.20		
Quality			
very low quality = 0.2			
low quality = 0.4			
normal quality = 0.6			
high quality = 0.8			
very high quality = 1.0			
Cost (\$/sq. meter)	10.00		
Interest Rate			
A.P.R. (%)	5%		
Tropical Stimuli			
time interval months	Ave temp °F	Ave. RH %	Chemical attack Intensity 0-1
1	94	92	0.27
2	89	93	0.27
3	93	94	0.26
			Degradation of Serviceability
			Repair Decision
			Durability Cost
			OK
			10
			OK
			0
			OK
			0

Figure 20. Behavior of paint cladding subjected to tropical stimuli (low-maintenance strategy).

## Paint Cladding

### INPUTS

		paint	Serviceability
Material			
Repair trigger	0.60	1.000	
Aesthetic level which triggers a repair			
Material/Maintenance	0.80		
Quality			
very low quality = 0.2			
low quality = 0.4			
normal quality = 0.6			
high quality = 0.8			
very high quality = 1.0			
Cost (\$/sq. meter)	10.00		
Interest Rate	5%		
A.P.R. (%)			



Tropical Stimuli  
50 year durability cost = \$10/sq. meter  
10 year durability cost = \$10/sq. meter

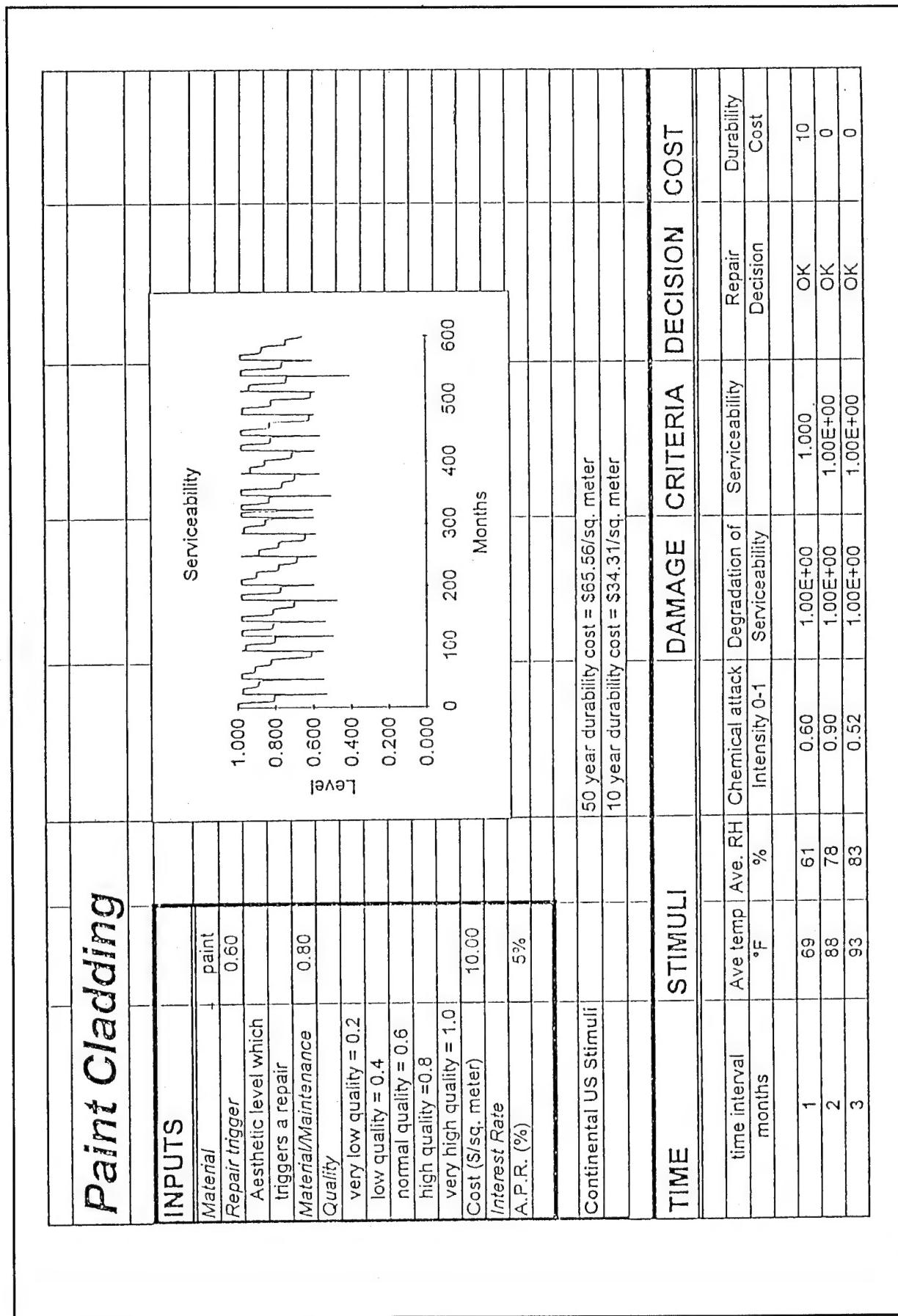
TIME	STIMULI	DAMAGE	CRITERIA	DECISION	COST
time interval months	Ave temp °F	Ave. RH %	Chemical attack Intensity 0-1	Degradation of Serviceability	Serviceability Repair Decision
1	87	92	0.17	1.00E+00	1.000 OK
2	94	95	0.11	1.00E+00	1.00E+00 OK
3	95	95	0.21	1.00E+00	1.00E+00 OK

Figure 21. Behavior of paint cladding subjected to tropical stimuli (high-maintenance strategy).

## Paint Cladding

INPUTS		Serviceability					
Material	paint	Level	1.000				
Repair trigger	0.60	0.800					
Aesthetic level which triggers a repair		0.600					
Material/Maintenance Quality	0.20	0.400					
very low quality = 0.2		0.200					
low quality = 0.4		0.000					
normal quality = 0.6							
high quality = 0.8							
very high quality = 1.0							
Cost (\$/sq. meter)	10.00	0	100				
Interest Rate		200	300				
A.P.R. (%)	5%	400	500				
Continental US Stimuli		600					
TIME	STIMULI	DAMAGE	CRITERIA DECISION COST				
time interval months	Ave Temp °F	Ave. RH %	Chemical attack Intensity 0-1	Degradation of Serviceability	Serviceability Decision	Repair Cost	Durability
1	59	69	0.49	1.00E+00	1.000	OK	10
2	85	74	0.51	9.94E-01	9.94E-01	OK	0
3	89	77	0.68	9.88E-01	9.88E-01	OK	0

Figure 22. Behavior of paint cladding subjected to continental U.S. stimuli (low-maintenance strategy).



**Figure 23.** Behavior of paint cladding subjected to continental U.S. stimuli (high-maintenance strategy).

## Aluminum Cladding

INPUTS		Serviceability			
Material	aluminum				
Cost (\$/sq. meter)	50.00				
Repair trigger	0.60				
Aesthetic level which triggers a repair					
Interest Rate A.P.R. (%)	5%				
Tropical Stimuli		50 year durability cost = \$71.56/sq. meter			
		10 year durability cost = \$50/sq. meter			
TIME		DAMAGE	DECISION	COST	
time interval months	Ave temp °F	Ave. RH %	Chemical attack Intensity 0-1	Serviceability	Repair Decision Durability Cost
1	88	92	0.17	1.000	OK \$50.00
2	95	94	0.30	9.98E-01	OK 0
3	98	94	0.24	9.97E-01	OK 0
4	92	94	0.28	9.95E-01	OK 0
5	90	92	0.12	9.94E-01	OK 0

Figure 24. Behavior of the aluminum cladding subjected to tropical stimuli.

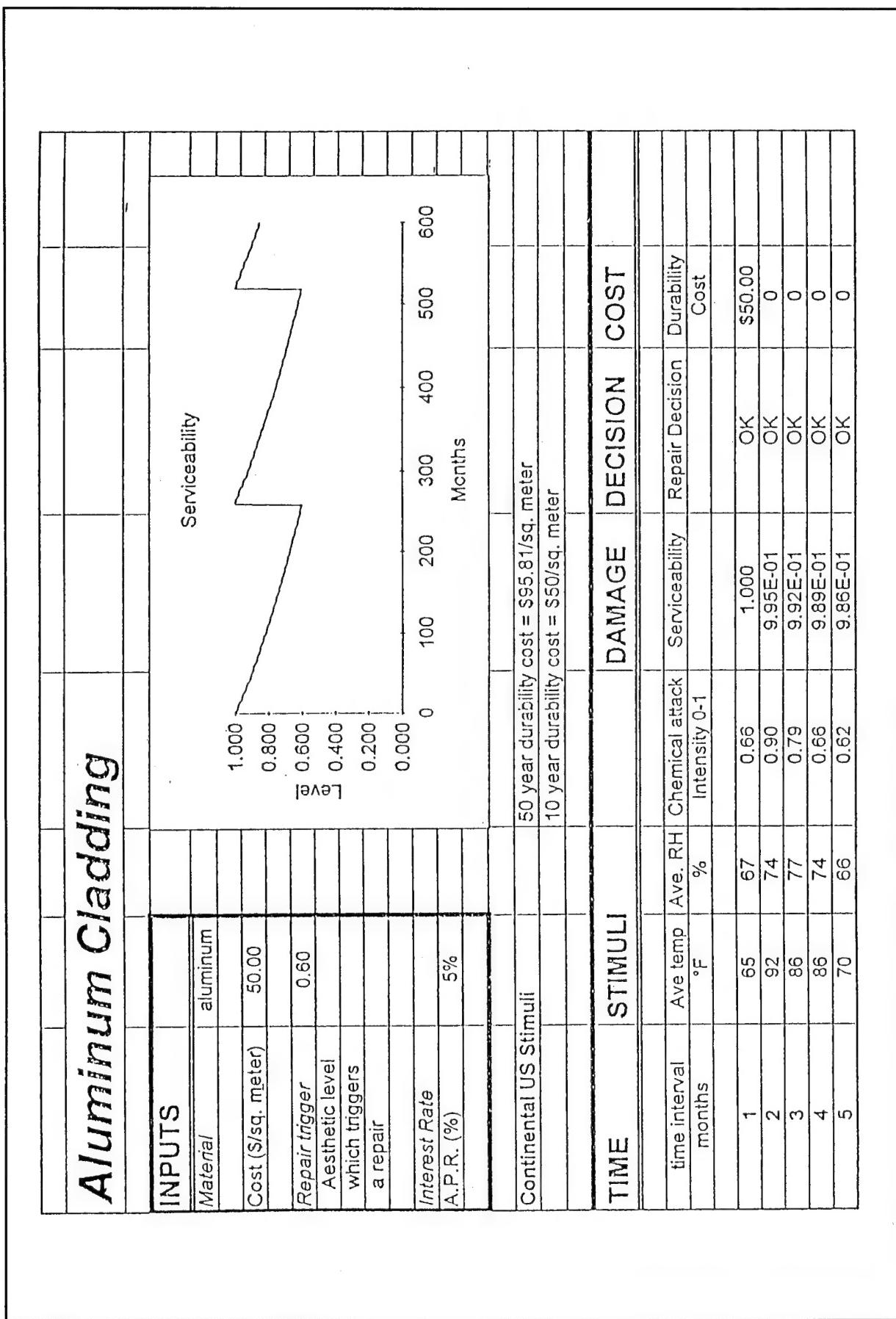


Figure 25. Behavior of aluminum cladding subjected to continental U.S. stimuli.

## 6 Conclusions and Recommendations

### Summary

A Building Materials Durability Model (BMDM) has been developed and implemented as a computer simulation to calculate the net present value of a building component as estimated from the component's first cost and the estimated cost of required future repairs. BMDM accepts arbitrary inputs of environmental stimuli, determines the durability for a token system having the nominal design requirements of the real structural component in service, considers simultaneous accumulation of damage due to multiple environmental phenomena, predicts the time to dysfunction or failure, and estimates total durability cost as the sum of first costs and the current cost of future repairs.

The simulations discussed in Chapter 5 show that BMDM can be used to:

- compare the life-cycle costs of two candidate materials for a given application
- assess the consequences of different repair strategies
- compare durability of a material in different environments (locations)
- choose the best durability strategy for short- and long-term applications
- consider the time cost of money in making durability strategy decisions.

It is concluded that BMDM could be used as the basis of a Monte Carlo simulation of a material's total durability cost and its uncertainty.

Work to date represents an initial development attempt and proof of concept.

### Recommendations

To fulfill its potential as a materials selection and costing tool for facility designers, BMDM requires refinements and extensions. It is recommended that future work on BMDM include the following:

- improving models for a given material and application using careful comparisons with observed behavior

- considering additional token systems
- surveying and cataloguing the durability problems for construction materials
- building up a library of damage models for needed applications
- increasing the sophistication of the environmental inputs and consider in more detail the relationship between measured environmental factors and the actual variation in these quantities experienced by the material
- altering the current model to permit the initial quality of the building material or component to be a random variable
- generalizing the framework for flexibility with respect to geometry and loading
- considering transport models (e.g., for moisture, chemicals and elements, heat, etc.) in more detail.

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